THE SECOND ASSIGNMENT.
Integration Theory 425.

This homework is due on Friday 15 September in class.
This homework will contribute to your grade.

MEASURABLE SETS.

1. For arbitrary sets $A_1, A_2, B_1, B_2$, is it true that
   \[(A_1 \cup A_2) \Delta (B_1 \cup B_2) \subset (A_1 \Delta B_1) \cup (A_2 \Delta B_2);\]
   \[(A_1 \cap A_2) \Delta (B_1 \cap B_2) \subset (A_1 \Delta B_1) \cap (A_2 \Delta B_2);\]
   \[(A_1 \setminus A_2) \Delta (B_1 \setminus B_2) \subset (A_1 \Delta B_1) \setminus (A_2 \Delta B_2)?\]

   **Reminder** An elementary set is, by definition, a finite union of rectangles.

2. Let $A, B \subset [0, 1]^2$ be elementary sets. Show that so are $A \cap B$, $A \cup B$, $A \setminus B$.

3. Let $E$ be an elementary set in the unit square. Show that $m([0, 1]^2 \setminus E) = 1 - m(E)$.

4. Let $A, B \subset [0, 1]^2$ be measurable sets. Show that so are $A \cap B$, $A \cup B$, $A \setminus B$.

5. Let $A_1, \ldots, A_n$ be arbitrary sets. Show that $A_1 \cup \ldots A_n = A'_1 \cup A'_2 \cdots \cup A'_n$, where $A'_i = A_i \setminus (A_1 \cup \cdots \cup A_{i-1})$ for $i > 1$ and $A'_1 = A_1$.

6. Let $E \subset [0, 1]^2$ be a set such that $m^*(E) + m^*([0, 1]^2 \setminus E) = 1$. Prove that $E$ is measurable.

   **REVIEW OF COMPLETENESS AND COMPACTNESS.**

   We define the real numbers as an ordered field additionally satisfying the supremum axiom.

7. Prove the Archimedean Property of the real numbers.

8. Prove that every monotonic bounded sequence of real numbers converges.

9. Prove the Principle of Nested Intervals for the real numbers.

10. Prove that every Cauchy sequence of real numbers converges.

11. Prove that every bounded sequence of real numbers has a convergent subsequence.

12. Prove that if the unit interval $[0, 1]$ is covered by countably many open intervals $I_i = (a_i, b_i)$, then there exists $N$ such that $I_1, \ldots, I_N$ cover $[0, 1]$. 