THE SEVENTH ASSIGNMENT.
Integration Theory 425.

This homework is due on Monday 6 November in class.
This homework will contribute to your grade.

FOURIER SERIES.

Compute the Fourier series for the functions defined below. The functions, introduced on $[0, 2\pi]$ or $[-\pi, \pi]$, are assumed to be $2\pi$-periodic on the whole line.

1. $f(x) = \frac{1}{2}(\pi - x)$ for $0 < x < 2\pi$, $f(0) = f(2\pi) = 0$.

2. $s(x) = 1$ if $0 < x < \pi$, $s(x) = -1$ if $-\pi < x < 0$, $s(-\pi) = s(\pi) = 0$.

3. $\xi(x) = 1$ if $x \in (-h, h)$; $\xi(x) = 0$ at the remaining points of $[-\pi, \pi]$ (here, of course, $h < \pi$).

4. Let $h \leq \frac{\pi}{2}$. The triangular function $\lambda_h$, defined on $[-\pi, \pi]$, is even and continuous. Furthermore, $\lambda(0) = 1$, $\lambda(x) = 0$ for $x \in [2h, \pi]$ and $\lambda$ is linear on $(0, 2h)$.

5. Take parameters $k, h$ in such a way that $0 < k \leq h$, $k + h \leq \pi$. The trapezoidal function $\mu_{k,h}$ is periodic, continuous, even, equal to 1 in $(0, h-k)$, 0 in $(h+k, \pi)$ and linear in $(h-k, h+k)$.

6. Show that $\mu_{h,k} = \frac{1}{2}(\frac{h+k}{k})\lambda_{(h+k)/2} - \frac{1}{2}(\frac{h+k}{k})\lambda_{(h-k)/2}$.

THE Riemann AND THE Lebesgue INTEGRALS.

7. Let $f : [0, 1] \to \mathbb{R}$ be Riemann integrable. Prove that $f$ is also Lebesgue integrable and the value of the integrals are the same.

8. Let $f : [0, 1] \to \mathbb{R}$ be Riemann integrable. Prove that the set of discontinuities of $f$ has measure zero.

9. Prove the converse: if a bounded function $f : [0, 1] \to \mathbb{R}$ is continuous on a set of full measure, then $f$ is Riemann integrable.

The characterization given by the two last problems is known as the Theorem of Riemann-Du Bois Reymond-Lebesgue.