Three Wedge Product Formulas

Suppose W, X, Y, Z are four vectors in a 2 dimensional subspace V of $\mathbf{R}^3$.

$$|X \wedge Y|^2 + <X, Y>^2 \;=\; |X|^2 |Y|^2 \;.$$

$$<W \wedge X, Y \wedge Z> \;=\; <W, Y><X, Z> + <W, Z><X, Y> \;.$$

$$L(X) \wedge L(Y) \;=\; (\det L) X \wedge Y \quad \text{for a linear } L : V \to V \;.$$

To verify these one may assume that $V = \mathbf{R}^2$ and compute using the formulas

$$< (a, b), (c, d) >= ab + cd \quad \text{and} \quad (a, b) \wedge (c, d) = ad - bc \;.$$