**Sub-super correction**

Recall our proof that $\Phi \circ u$ *is a subsolution whenever* $u$ *is a subsolution and* $\Phi$ *is convex and increasing.* In case $\Phi$ is $\mathcal{C}^2$, we use the chain rule and product rule to get

$$\sum_{i,j} \int_{\mathbf{B}} a_{ij} D_i \big(\Phi \circ u\big) D_j \phi \;=\; \sum_{i,j} \int_B a_{ij} \Phi'(u) D_i u D_j \phi$$
$$= \sum_{i,j} \int_B a_{ij} D_i u D_j (\Phi'(u)\phi) \;-\; \sum_{i,j} \int_B (a_{ij} D_i u D_j u) \phi \Phi''(U) \;\leq\; 0 \;.$$

The negativity of the 2nd term follows from the positivity of $\phi$, of $\Phi''$, and of the matrix $a_{ij}$. The negativity of the first term used that $\Phi$ was *increasing*, that $\phi$ is positive, and that $u$ is a *sub*solution.

In case $u$ is a *super*solution and $F$ is convex and *decreasing*, the same inequality shows that $\Phi \circ u$ is again a *sub*solution.

This is the case we use with $\Phi(s) = (\log s)^-$.