Undergraduate Lecture:

Title: The Number Theory of Partitions: The Legacy of Euler, Freeman Dyson, and Ramanujan.

Abstract: At first glance the stuff of partitions seems like child's play:

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1.$$

Therefore, there are 5 partitions of the number 4. But (as happens in Number Theory) the seemingly simple business of counting the ways to break a number into parts leads quickly to some difficult and beautiful problems. Partitions play important roles in such diverse areas of mathematics as Combinatorics, Lie Theory, Representation Theory, Mathematical Physics, and the theory of Special Functions, but we shall concentrate here on their role in Number Theory. We shall give an account of the impact of Euler, Dyson and Ramanujan on the subject, and describe some of the recent advances in the subject.

Lecture 1:

Title: Modular form congruences.

Abstract. Modular forms and functions provide a large supply of generating functions whose coefficients contain valuable Number Theoretic information. These include problems related to quadratic forms, elliptic curves over finite fields, values of L-functions, and partitions. Here we survey these varied roles and give a snapshot regarding the congruence properties of these objects. These examples culminate with recent work extending classical observations of Ramanujan on partitions.

Lecture 2:

Title: Groups associated to quadratic fields and elliptic curves.

Abstract. Ideal class groups of number fields play many roles in Number Theory. They are used in studying Diophantine equations (e.g. Fermat's Last Theorem), in class field theory, and arithmetic geometry. Elliptic curves also provide a supply of analogous groups such the Selmer groups, the Shafarevich-Tate groups, and the Mordell-Weil groups. Here we shall give an account of old and new results on famous conjectures for these groups. These include conjectures of Gauss, Cohen and Lenstra, and Goldfeld.

Lecture 3:

Title: Bizarre q-series and values of L-functions.

Abstract. The values of the Riemann zeta-function at positive even integers are given by the ordinary Bernoulli numbers. It is not so well known that these values are interpolated by 'very strange' q-series. Here we extend this classical fact by presenting several families of q-series whose specializations give general values of various L-functions. These series can be of basic hypergeometric type, and they can also be limiting sums of truncations of Borcherds products. This lecture will survey these results beginning with classical identities of Euler and Jacobi, and culminate with recent findings.