

# Factorizations of Algebraic Integers, Block Monoids, and Additive Number Theory

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## **Abstract**

Let  $D$  be the ring of integers in a finite extension of the rationals. The classic examination of the factorization properties of algebraic integers usually begins with the study of norms. In this paper, we show using the ideal class group,  $\mathcal{C}(D)$ , of  $D$  that a deeper examination of such properties is possible. Using the class group, we construct an object known as a block monoid, which allows us to offer proofs of three major results from the theory of nonunique factorizations: Geroldinger's theorem, Carlitz's theorem and Valenza's theorem. The combinatorial properties of block monoids offer a glimpse into two widely studied constants from additive number theory, the Davenport constant and the cross number. Moreover, block monoids allow us to extend these results to the more general classes of Dedekind domains and Krull domains.