

Introduction to Riemannian holonomy groups and calibrated geometry

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Abstract: The holonomy group of a Riemannian manifold (M, g) is a global invariant which encodes the tensors constant under the Levi-Civita connection of g . The possible holonomy groups were classified by Berger in 1955, and include Kahler, hyperkahler, Calabi-Yau manifolds and the exceptional geometries G_2 and $Spin(7)$ – basically, a list of the most interesting special geometric structures in Riemannian geometry.

Calibrated geometry was introduced by Harvey and Lawson in 1981. Given a Riemannian manifold (M, g) , one uses a closed k -form ϕ called a calibration to define a distinguished class of minimal k -dimensional submanifolds N in M called calibrated submanifolds. It is a natural companion to Riemannian holonomy groups, because manifolds with special holonomy come equipped with one or more natural, interesting calibrations. Examples include complex submanifolds of Kahler manifolds and special Lagrangian m -folds in Calabi-Yau m -folds.

This will be an introductory survey at an elementary level, with no new research. Experts should skip it and start early on the buffet.