On Robustness Against Misspecified Mixing Distribution in Generalized Linear Mixed Models

The generalized linear mixed model (GLMM) extends classical regression analysis to non-normal, correlated response data. Our focus is primarily on mixed Poisson regression models, a class of GLMM’s which is commonly used to analyze count data that exhibit overdispersion. Because inference for these models can be computationally difficult, simplifying distributional assumptions are often made. Using an influence function approach, we examine the robustness of parameter estimates when a main component of the model, the mixing distribution, is misspecified. Results for maximum likelihood, quasi-likelihood, and hierarchical likelihood estimates are presented.

We define an influence function representing effects of infinitesimal perturbations of the mixing distribution. This enables us to compute Gâteaux derivatives of functionals (estimates) under perturbations of the mixing distribution for maximum likelihood estimates (MLE’s), assuming Poisson–gamma, Poisson–inverse Gaussian, and Poisson–lognormal models. Provided the first two moments exist, these MLE’s are robust in the sense that their Gâteaux derivatives are bounded.

A limited simulation study of misspecified Poisson–gamma regression models shows that estimated regression coefficients are very robust, but estimates of the random effect are only robust under small perturbations of the nominal model.

The influence function of quasi-likelihood estimates is also derived, and these estimates are shown to be robust. Maximum hierarchical likelihood estimates, on the other hand, may not even be consistent, as shown by a specific counterexample.