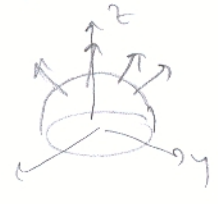
Homework 10 Solutions

Math 212 - April 15, 2009

Section 8.2

1. Use Stokes' theorem to evaluate $\iint_S (\nabla\times F)\cdot dS$

where $S =$  and $F(x,y,z) = (y, -x, zx^3y^2)$.

$$\iint_S (\nabla\times F)\cdot dS = \int_{\partial S} F\cdot ds = \int_0^{2\pi} F(c(t))\cdot c'(t)\,dt$$

$c(t) = (\cos t, \sin t, 0)$

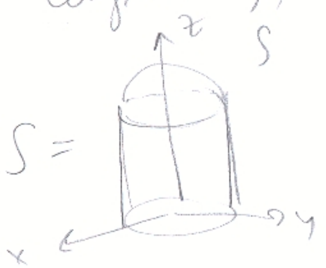
$c'(t) = (-\sin t, \cos t, 0)$

$$= \int_0^{2\pi} (\sin t, -\cos t, 0)\cdot(-\sin t, \cos t, 0)\,dt$$

$$= \int_0^{2\pi} -\sin^2 t - \cos^2 t\,dt$$

$$= -\int_0^{2\pi} dt = \boxed{-2\pi}$$

5. Compute $\iint_S (\nabla\times F)\cdot dS$ where

$S =$  and $F(x,y,z) = (zx + z^2y + x, z^3yx + y, z^4x^2)$.

$$\iint_S (\nabla\times F)\cdot dS = \int_{\partial S} F\cdot ds = \int_0^{2\pi} F(c(t))\cdot c'(t)\,dt$$

$c(t) = (\cos t, \sin t, 0)$

$c'(t) = (-\sin t, \cos t, 0)$

$$= \int_0^{2\pi} (\cos t, 0, 0)\cdot(-\sin t, \cos t, 0)\,dt$$

$$= \int_0^{2\pi} -\cos t\sin t\,dt$$

$= 0$. (orientation doesn't matter.)