

Homework 2 solutions

Math 212

January 21, 2009

Section 1.3

1. Verify that interchanging the first two rows changes the sign of the determinant.

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 2 & 0 & 2 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix}$$

$$= 0 - 2(3 \cdot 2 - 2 \cdot 1) + 0$$

$$= -8$$

$$\curvearrowright \begin{vmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 0 & 2 \end{vmatrix} = 3 \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} - 0 \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}$$

$$= 3(2 \cdot 2 - 0 \cdot 1) - 0 + 1 \cdot (1 \cdot 0 - 2 \cdot 2)$$

$$= 12 - 4 = 8$$

Yes, $-8 + 8$ differ by multiplying by -1 .

7. What is the volume of the parallelepiped with sides $2i + j - k$, $5i - 3k$, and $i - 2j + k$?

$$\pm(\text{Volume}) = \begin{vmatrix} 2 & 1 & -1 \\ 5 & 0 & -3 \\ 1 & -2 & 1 \end{vmatrix} = 2 \begin{vmatrix} 0 & -3 \\ -2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 5 & -3 \\ 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 5 & 0 \\ 1 & -2 \end{vmatrix}$$

$$= 2(0 - 6) - 1(5 + 3) - 1(-10 - 0)$$

$$= -12 - 8 + 10$$

$$= -10$$

So the volume is 10.