Homework 5 Solutions

Math 212 — Feb. 25 2009

Section 3.1 4. $f(x,y) = e^{-xy^2} + y^3x^4$

$\frac{\partial f}{\partial x} = -y^2e^{-xy^2} + 4y^3x^3$ $\frac{\partial f}{\partial y} = -2xye^{-xy^2} + 3y^2x^4$

$\frac{\partial^2 f}{\partial x^2} = y^4e^{-xy^2} + 12y^3x^2$ $\frac{\partial^2 f}{\partial x \partial y} = -2ye^{-xy^2} + 2xy^3e^{-xy^2} + 12y^2x^3$

$\frac{\partial^2 f}{\partial y \partial x} = -2ye^{-xy^2} + (2xy)y^2e^{-xy^2} + 12y^2x^3$

The mixed partials agree.

$\frac{\partial^2 f}{\partial y^2} = -2xe^{-xy^2} + (2xy)2xye^{-xy^2} + 6yx^4.$

Section 3.3 Find the critical points & analyze local nature.

5. $f(x,y) = e^{1+x^2-y^2}$

$Df = [2xe^{1+x^2-y^2} \quad -2ye^{1+x^2-y^2}] = [0 \quad 0].$

The only critical point is $(0,0)$.

$f_{xx} = 2e^{1+x^2-y^2} + (2x)^2e^{1+x^2-y^2}$ $f_{xx}(0,0) = 2e$

$f_{yy} = -2e^{1+x^2-y^2} + (2y)^2e^{1+x^2-y^2}$ $f_{yy}(0,0) = -2e$

$f_{xy} = -2y \cdot 2x\, e^{1+x^2-y^2}$ $f_{xy}(0,0) = 0$

$D = f_{xx}f_{yy} - (f_{xy})^2 = 2e \cdot (-2e) - 0^2 = -4e^2.$

By the second derivative test, the graph of f has a saddle point at $(0,0)$.