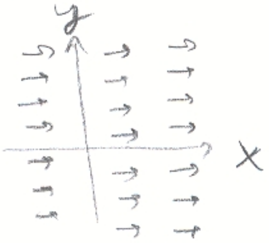
Homework 6 Solutions

Math 212 — March 11, 2009

Section 4.3

2. Sketch the vector field $F(x,y)=(4,0)$.

Do NOT GRADE.



This problem was not really assigned. problems 16, 18 ~~typo~~

16. Show $c(t)=\left(\frac{1}{t^3}, e^t, \frac{1}{t}\right)$ is a flow line of the vector field $F(x,y,z)=(-3z^4, y, -z^2)$.

$c'(t)=\left(-\frac{3}{t^4}, e^t, -\frac{1}{t^2}\right)$.

$F(c(t))=\left(-3\cdot\left(\frac{1}{t}\right)^4, e^t, -\left(\frac{1}{t}\right)^2\right)$

So $c'(t)=F(c(t))$ and $c(t)$ is a flow line.

18. Let $c(t)$ be a flow line of $F=-\nabla V$. Prove that $V(c(t))$ is a decreasing function of t.

$$\frac{d}{dt}V(c(t)) = \nabla V(c(t))\cdot c'(t)$$
$$= -F(c(t))\cdot c'(t)$$
$$= -F(c(t))\cdot F(c(t))$$
$$\leq 0.$$

The t derivative is ≤ 0, So $V \circ c$ is decreasing.

Section 4.4

1. Find the divergence of $V(x,y,z)=e^{xy}i - e^{xy}j + e^{yz}k$

$$\nabla\cdot V = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(e^{xy}, -e^{xy}, e^{yz}\right) = ye^{xy} - xe^{xy} + ye^{yz}.$$