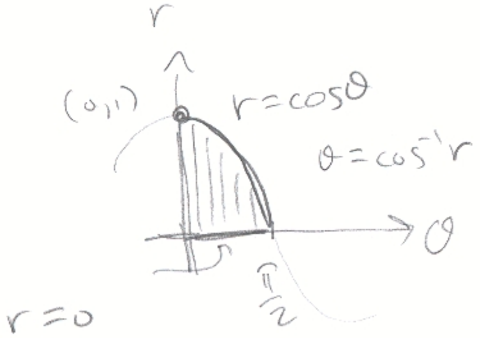
Homework 7 Solutions

Math 212 — March 18, 2009

Section 5.4

1b): $$\int_0^{\pi/2}\int_0^{\cos\theta} \cos\theta \, dr\, d\theta = \int_0^{\pi/2} \cos\theta \, r \Big|_{r=0}^{\cos\theta} d\theta$$

$$= \int_0^{\pi/2} \cos^2\theta \, d\theta$$



$$= \int_0^{\pi/2} \frac{1+\cos 2\theta}{2} \, d\theta$$

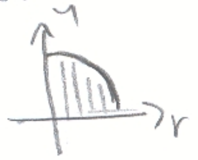
$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4} \Big|_0^{\pi/2}$$

$$= \frac{\pi}{4}.$$

$$\int_0^1 \int_0^{\cos^{-1} r} \cos\theta \, d\theta \, dr = \int_0^1 \sin\theta \Big|_{\theta=0}^{\theta=\cos^{-1} r} dr$$

$$= \int_0^1 \sin(\cos^{-1} r) \, dr$$

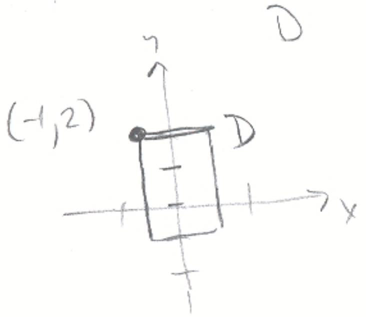
$$= \int_0^1 \sqrt{1-r^2} \, dr$$

= Area of  $\frac{1}{4}$ of unit disk

$$= \frac{\pi}{4}.$$

5. If $D = [-1, 1] \times [-1, 2]$, show that

$$1 \leq \iint_D \frac{dx\,dy}{x^2+y^2+1} \leq 6.$$



In the region D, $0 \leq x^2+y^2 \leq 1^2+2^2 = 5$

So $1 \leq x^2+y^2+1 \leq 6$

& $1 \geq \frac{1}{x^2+y^2+1} \geq \frac{1}{6}.$

So $$\frac{1}{6} \cdot \text{Area}(D) \leq \iint_D \frac{dx\,dy}{x^2+y^2+1} \leq 1 \cdot \text{Area}(D).$$

Since $\text{Area}(D) = 6$, we have

$$1 \leq \iint_D \frac{dx\,dy}{x^2+y^2+1} \leq 6.$$

<u>Section 5.5</u>

4. $$\iiint_B z e^{x+y}\,dx\,dy\,dz = \int_0^1\int_0^1\int_0^1 z e^{x+y}\,dx\,dy\,dz$$

$B = [0,1] \times [0,1] \times [0,1]$

$$= \int_0^1\int_0^1 z e^{x+y}\Big|_{x=0}^{1}\,dy\,dz$$

$$= \int_0^1\int_0^1 (z e^{y+1} - z e^{y})\,dy\,dz$$

$$= \int_0^1 z e^{y+1} - z e^{y}\Big|_{y=0}^{1}\,dz$$

$$= \int_0^1 z(e^2 - e - (e-1))\,dz = \frac{e^2-2e-1}{2}$$