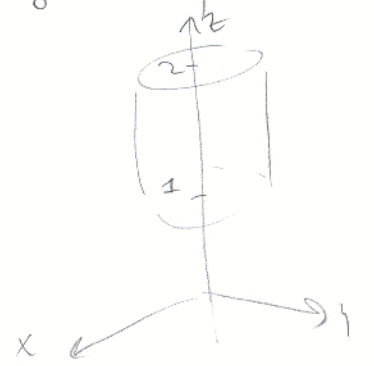
Homework 8 Solutions
Math 212 – April 1, 2009

Section 6.3

10. Find the center of mass of the cylinder W $x^2+y^2\le 1$, $1\le z\le 2$, with density $\delta=(x^2+y^2)z^2$.



By symmetry, $\bar{x}=0$ and $\bar{y}=0$.

$$\bar{z}=\frac{\iiint_W z\delta\,dx\,dy\,dz}{\iiint_W \delta\,dx\,dy\,dz}$$

$$\iiint_W z(x^2+y^2)z^2\,dx\,dy\,dz=\int_1^2\int_0^{2\pi}\int_0^1 r^2z^3\cdot r\,dr\,d\theta\,dz \quad \text{in cylindrical coordinates}$$

$$=2\pi\cdot\frac{r^4}{4}\Big|_0^1\cdot\frac{z^4}{4}\Big|_1^2$$

$$=2\pi\cdot\frac{1}{4}\cdot\frac{16-1}{4}=\frac{15\pi}{8}$$

$$\iiint_W (x^2+y^2)z^2\,dx\,dy\,dz=\int_1^2\int_0^{2\pi}\int_0^1 r^2\cdot z^2\cdot r\,dr\,d\theta\,dz$$

$$=2\pi\cdot\frac{r^4}{4}\Big|_0^1\cdot\frac{z^3}{3}\Big|_1^2$$

$$=2\pi\cdot\frac{1}{4}\cdot\frac{8-1}{3}=\frac{7\pi}{6}$$

$$\bar{z}=\frac{\frac{15\pi}{8}}{\frac{7\pi}{6}}=\frac{15\cdot 3}{7\cdot 4}=\frac{45}{28}$$

Center of mass $=\left(0,0,\frac{45}{28}\right)$