Homework 9 solutions

Math 212 — April 8, 2009

section 7.3

2. Find an equation for the plane tangent to $x=u^2-v^2$, $y=u+v$, $z=u^2+4v$ at $\left(-\frac{1}{4}, \frac{1}{2}, 2\right)$.

$T_u = (2u, 1, 2u)$

$T_v = (-2v, 1, 4)$

$$T_u \times T_v = \begin{vmatrix} i & j & k \\ 2u & 1 & 2u \\ -2v & 1 & 4 \end{vmatrix} = (4-2u, -(8u+4uv), 2u+2v)$$

$$\left.\begin{aligned} u+v &= \tfrac{1}{2} \\ u^2-v^2 = (u+v)(u-v) &= -\tfrac{1}{4} \\ \tfrac{1}{2}(u-v) &= -\tfrac{1}{4} \\ u-v &= -\tfrac{1}{2} \end{aligned}\right\} \rightarrow \quad u=0 \ \& \ v=\tfrac{1}{2}.$$

$$T_u \times T_v\left(0, \tfrac{1}{2}\right) = (4, 0, 1),$$

$$4\left(x+\tfrac{1}{4}\right)+0\left(y-\tfrac{1}{2}\right)+1(z-2)=0.$$

$$\boxed{4x+z=1}$$

6. Find a unit vector normal to $x=3\cos\theta\sin\phi$, $y=2\sin\theta\sin\phi$, $z=\cos\phi$ for θ in $[0,2\pi]$ and ϕ in $[0,\pi]$.

This is an ellipsoid.

$T_\theta = (-3\sin\theta\sin\phi, 2\cos\theta\sin\phi, 0)$

$T_\phi = (3\cos\theta\cos\phi, 2\sin\theta\cos\phi, -\sin\phi)$

$$T_\theta \times T_\phi = \begin{vmatrix} i & j & k \\ -3\sin\theta\sin\phi & 2\cos\theta\sin\phi & 0 \\ 3\cos\theta\cos\phi & 2\sin\theta\cos\phi & -\sin\phi \end{vmatrix} = (-2\cos\theta\sin^2\phi, -(3\sin\theta\sin^2\phi), -6\sin\phi\cos\phi)$$

$$\frac{T_\theta \times T_\phi}{\|T_\theta \times T_\phi\|} = \frac{(2\cos\theta\sin\phi, 3\sin\theta\sin\phi, 6\cos\phi)}{\sqrt{4\cos^2\theta\sin^2\phi + 9\sin^2\theta\sin^2\phi + 36\cos^2\phi}}.$$