Hm #2- Solutions

(2.7/2.2) 9 points

If $N(t)$ is the amount of ^{131}I at time t,
then $N'(t) = -\lambda N(t)$. (from lecture)

Hence $N(t) = N_0 e^{-\lambda t}$, N_0 - initial amount.

$N(0) = \boxed{N_0 = 500\,mg}$

$T_{1/2} = \frac{\ln 2}{\lambda}$ => $\boxed{\lambda} = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{8.04} = \boxed{0.0862}$

how to get $T_{1/2}$

We have $\boxed{N(t) = 500\, e^{-0.0862 t}}$

Thus, $\boxed{N(20)} = 500\, e^{-0.0862 \cdot 20} = \boxed{89.153\,mg}$.

$T_{1/2}$ is such that $N(T_{1/2}) = 250\,mg$, i.e.

$$250 = 500\, e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$
$$\Rightarrow \ln 1/2 = -\lambda T_{1/2}$$
$$\Rightarrow -\ln 2 = -\lambda T_{1/2}$$
$$\Rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda}}$$

(33/2.2) 9 points Let $T(t)$ be the temperature of the body at time t.

From we know $\boxed{T(t) = A + (T(0) - A)\, e^{-kt}}$

$T(0)$ - initial temperature
A - ambient temperature

Assume that midnight is $t = 0$; $\boxed{T(0) = 31\,°C \;;\; A = 21\,°C \;;\; T(1) = 29\,°C}$

From $T(1) = 29\,°C$: $21 + (31 - 21)\, e^{-k} = 29 \Rightarrow \boxed{k = \ln \frac{10}{8} \approx 0.223}$

Hence $\boxed{T(t) = 21 + 10\, e^{-0.223 t}}$

Next we look for time t such that $T(t) = 37\,°C$:

$$21 + 10\, e^{-0.223 t} = 37 \Rightarrow \boxed{t} = -\frac{\ln \frac{10}{16}}{0.223} \approx -2.1 \text{ hours} \approx \boxed{-2 \text{ hours and } 6 \text{ minutes}}$$