373K Algebra I, Homework 8

From Artin

Chapter 6 (pp 190–191) 7.1, 7.2, 8.1, 8.2, 9.1, 9.2.

Others:

1. Suppose that a group G acts on a set X. Define $\sim$ on X by $x \sim y$ if and only if $y = g \cdot x$ for some $g \in G$. Prove that $\sim$ is an equivalence relation.

2. Compute the image of S_3 in S_6 under the left regular representation.

3.(i) Let G be a finite group and $\pi : G \to S_G$ the left regular representation. Assume that $|G| = mn$ with m and n co-prime. Suppose that $x \in G$ has order n, show that $\pi(x)$ is a product of m n-cycles in S_{mn}.

(ii) Suppose that p is a prime. Show that any group G of order $2p$ has a normal subgroup order p

4. Let $G = \mathbf{R}^*$ (the multiplicative group of the non-zero real numbers), and let $\mathbf{R}_+$ denote the set of positive real numbers. Define a map $G \times \mathbf{R}_+ \to \mathbf{R}_+$ by $(r, a) \mapsto a^r$. Is this a G-action?

5. How many permutations in S_8 commute with $(1\ 3\ 5)(2\ 4)(6\ 7)$?