

Instructor: Prof. Anthony Várilly-Alvarado **Time:** MWF 2:00-2:50PM
Office: 222 Herman Brown **Classroom:** Herzstein Hall 210
Email: varilly@rice.edu **Office Hours:** W 5-6PM, Th 3-4PM
Class Webpage: Look for Math 354 001 F17 on Canvas.

Teaching Assistants: Dan Bernazzani and Sarah Seger. Recitations and Office hours: TBA.

Prerequisites: A 200-level Math course is recommended. Some exposure to linear algebra at the level of Math 211 or 221/222 is strongly recommended, though not strictly necessary (Math 221 and 354 can be taken concurrently and they complement each other well). The only real prerequisite is a willingness to think hard about abstract mathematics, and to spend time grappling with ideas that will change your life. Talk to me if you are unsure whether you are ready to take this class.

Text: Sheldon Axler, *Linear Algebra Done Right* (Third edition). Undergraduate Texts in Mathematics, Springer, New York 2015. ISBN: 978-3-319-11079-0.

Videos: Professor Axler has recorded videos to accompany the book. We will use these to complement the lectures: <http://www.linear.axler.net/LADRvideos.html>

Homework: Due once a week, on **Friday, at 5pm** in my office. You are welcome (in fact, encouraged) to hand in the homework at the beginning of lecture on Friday.

The homework is not pledged and you can collaborate with other students in the class. In fact, you are very much *encouraged* to do so. However, you are not allowed to look up solutions in any written form; in particular, you are not allowed to look up solutions online. Students caught violating this rule will be reported to the Honor Council. You should write up your solutions individually.

Homework is a very important component of the course. This class has a heavy workload, and you should expect to spend a lot of time doing homework. Math 354 is in many ways similar to a language course: you must get lots of hands-on practice to internalize the definitions.

Exams: There will be two midterms, on **Wednesday, September 27th** and on **Wednesday, November 1st**. These exams will take place 7:00–9:00pm. If you have a conflict with these times, let me know about it in the first two weeks of classes. There will also be a written, 3-hour final exam.

Final exam: The date for the final exam is not available at this time. It is the policy of the Mathematics Department that no final may be given early to accommodate student travel plans. If you make travel plans that later turn out to conflict with the scheduled exam, then it is your responsibility to either reschedule your travel plans or take a zero in the final.

If an exam conflicts with a holiday you observe, please let me know.

Grades: Homework will count for 30% of your final grade; your lowest homework score will be dropped. Each midterm will count for 20% of your grade (for a total of 40%) and the final exam will count for 30% of your grade.

Attendance: Attendance is not required. However, you are responsible for all the material and announcements covered in lecture. While Canvas is a valuable resource, not all announcements will be posted there. Nevertheless, you are responsible for reading any emails I send to the class through Canvas.

Expectations: In my experience as a student, most people do not follow all the details of a Math lecture in real time. During lecture, you should expect to witness the big picture of what's going on. You should pay attention to the lecturer's advice on what is important and what isn't. A lecturer spends a long time thinking about how to deliver a presentation of an immense amount of material; they do not expect you to follow every step, but they do expect you to go home and fill in the gaps in your understanding. Not attending lecture really hurts your chances at a deep understanding of the material.

Disability Support: Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.

Topics to be covered

I plan to cover most of Chapters 1-8 of Axler's book, supplemented by some applications to show you the power of linear algebra within science, e.g., by providing precise language to explain Heisenberg's uncertainty principle, and in real life, e.g., by looking at Google's PageRank algorithm, which sorts out webpages according to their relative importance, and also at a basic image compression algorithm. This will require us to cover Chapter 4 of the book quickly.

1. **Vector Spaces and linear maps:** Definitions and basic properties.; linear independence, span, bases, dimension. Rank-nullity theorem. Quotient spaces and duality.
2. **Subspaces and eigenvalues:** Invariant subspaces and existence of eigenvalues.
3. **Inner product spaces:** Orthonormal bases and the Gram-Schmidt algorithm.
4. **The Spectral Theorem:** Orthonormal bases of eigenvectors. Singular Value Decomposition.
5. **Minimal + characteristic polynomials:** Generalized eigenvectors; Jordan canonical form.
6. (Time permitting) **Trace and Determinant:** Basics and properties.
7. **Applications:** Google PageRank, image compression, and the Heisenberg Uncertainty Principle (including the basics of axiomatic quantum mechanics).

Learning Outcomes: At the end of this course you should:

1. Be able to follow and produce a variety of short mathematical proofs, including proofs by application of definitions, by induction, by contradiction, by contrapositive, etc.
2. Be familiar with the core concepts of abstract linear algebra, including vector spaces, linear transformations (including kernels and images), duality, invariant subspaces and (generalized) eigenvalues, inner product spaces, orthogonality, self-adjoint, normal, and positive operators, characteristic and minimal polynomials.
3. Be able to state, understand, and apply structural theorems from abstract linear algebra to solve problems of a theoretical nature. Examples of these theorems include invertibility criteria, diagonalizability criteria and direct sum decompositions, Gram-Schmidt orthonormalization, the Spectral Theorem, Jordan block decompositions.
4. Understand how abstract linear algebra underpins some everyday computational tasks invisible to the naked eye, e.g., internet search results, image compression, etc.

Tentative Assignment/Exam Schedule:

September 1st: Problem Set #1 due.
September 8th: Problem Set #2 due.
September 15th: Problem Set #3 due.
September 22nd: Problem Set #4 due.

September 27th: Midterm 1; 7:00–9:00pm.

October 6th: Problem Set #5 due.
October 13th: Problem Set #6 due.
October 20th: Problem Set #7 due.
October 27th: Problem Set #8 due.

November 1st: Midterm 2; 7:00–9:00pm.

November 10th: Problem Set #9 due.
November 17th: Problem Set #10 due.
December 1st: Problem Set #11 due.

Final Exam: Date and time TBA by the registrar's office (usually on Week 8).