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| **Instructor:** | Prof. Anthony Várilly-Alvarado | **Time:** | MWF 10:00-10:50AM |
| **Office:** | 222 Herman Brown | **Classroom:** | Herman Brown Hall 227 |
| **Email:** | `varilly@rice.edu` | **Office Hours:** | M 3-4PM, Tu 3-4PM |

**Class Webpage:** Look for `Math 376 001 F17` on Canvas.

**Teaching Assistants:**

Graduate TA: Stephen Wolff. Recitation: TBA.
Communication Assignment Support: Dr. Elizabeth Festa (PWC) and Dr. Neil Fullarton (MATH).

**Prerequisites:** Officially, the prerequisites for this course are: an upper level course in Linear Algebra (Math 354 or Math 355), as well as exposure to Abstract Algebra (Math 306 or Math 356). This year, I will not enforce the latter prerequisite, because the book we will follow does not require knowledge of group theory.

I expect you to know the rudiments of mathematical proofs (e.g., mathematical induction, proof by contradiction, proof by contrapositive, negating quantifiers, etc). Talk to me if you are unsure whether you are ready to take this class.

**Text:** Cox, Little, O'Shea, *Ideals, Varieties, and Algorithms* (Fourth edition). Undergraduate Texts in Mathematics, Springer, 2015. ISBN: 978-3-319-16720-6.

**Homework:** Due once a week, on **Wednesday, at 5pm** in my office. You are welcome (in fact, encouraged) to hand in the homework at the beginning of lecture on Wednesday.

The homework is not pledged and you can collaborate with other students in the class. In fact, you are very much *encouraged* to do so. However, you are not allowed to look up solutions in any written form; in particular, you are not allowed to look up solutions online. Students caught violating this rule will be reported to the Honor Council. You should write up your solutions individually.

Homework is a very important component of the course. This class has a heavy workload, and you should expect to spend a lot of time doing homework and working on your final presentation.

**Midterm Exams:** There will be two in-class midterms, on **Friday, September 29th** and on **Wednesday, November 15th**. If an exam conflicts with a holiday you observe, please let me know within the first two weeks of class.

**Final Presentation:** In lieu of a final exam, each student will do a 25 minute slide presentation during the 3-hour time slot assigned by the Registrar's Office during Final Exams Week.

**Rationale:** Whether you are going into industry, or to graduate school in math, sciences, or for a professional degree, being able to give cogent oral presentations that summarize the motivation and key *ideas* behind a theorem, an algorithm, or (more broadly) a strategy can help you advance a point of view, an agenda, and your career. We have partnered up this semester with the Program in Writing and Communication to help you develop this skill.

Deathly afraid of public speaking? So was I at some point in my life. It's not a problem: effective communication is something that can be learned, but you can only get better at it by practicing!

Each student in the course will give a 25 minute presentation on a topic to be selected from a list that will be made available in the next few weeks, but which will include:

- Reed-Solomon codes (data transmission).
- Factoring integers using elliptic curves.
- Mixed volumes and Bernstein's theorem.
- The 27 lines on a complex cubic surface.

The presentations will use slides, and they will be accompanied by a short abstract (final version: 200 words). You are welcome to use LaTeX, Power Point, or Keynote in your presentation; in all cases you will need to learn a little bit of LaTeX to create good math displays. We do not expect you to know LaTeX going into the course, and Dr. Fullarton is going to provide resources to help you get started on this programming language.

The problem sets for the course will taper off in the last third of the semester in order to give you time and space to prepare your final presentation (see the tentative schedule below).

This assignment is worth 30% of your grade, divided as follows:

- Abstract (5%): 1% for completing the initial 350–400 version that outlines your talk, 1% for submitting a revised, shortened version (200 words) when you practice your presentation with a communication coach, and 3% for the final version of the abstract, to be submitted at the time of your final presentation.
- Presentation (25%): 2% for delivering a short slide presentation (2-3 slides; 5 minutes) to a small audience of your peers the week of October 30th; 3% for delivering a 10-minute mock presentation to a communication coach the week(end) just before Thanksgiving; 20% for the final presentation and slides.

The final presentation **will be video-taped**: per the Registrar's Office's instructions, all presentations have to occur in the 3-hour time slot assigned for a final exam for the course. With over 20 students enrolled in the course as of this writing, each one speaking for 25 minutes, the only way we can do this is to divide the class into small groups and have parallel sessions of final presentations. The purpose of recording the presentations is to give a chance for several instructors and communication coaches to assess your work.

> **Final presentation:** The date for a final presentation slot is not available at this time (it typically becomes available on Week 8). It is the policy of the Mathematics Department that no final exam/assignment may be given early to accommodate student travel plans. If you make travel plans that later turn out to conflict with the scheduled time slot, then it is your responsibility to either reschedule your travel plans or take a zero in the final presentation.

**Grades:** Homework will count for 40% of your final grade. Each midterm will count for 15% of your grade (for a total of 30%) and the final presentation will count for 30% of your grade.

**Attendance:** Attendance is not required. However, you are responsible for all the material and announcements covered in lecture. While Canvas is a valuable resource, not all announcements will be posted there. Nevertheless, you are responsible for reading any emails I send to the class through Canvas.

**Expectations:** In my experience as a student, most people do not follow all the details of a Math lecture in real time. During lecture, you should expect to witness the big picture of what's going on. You should pay attention to the lecturer's advice on what is important and what isn't. A lecturer spends a long time thinking about how to deliver a presentation of an immense amount of material; they do not expect you to follow every step, but they do expect you to go home and fill in the gaps in your understanding. Not attending lecture really hurts your chances at a deep understanding of the material.

**Disability Support:** Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.

## Topics to be covered

I plan to cover most of Chapters 1–4 and 8 of Cox, Little, and O'Shea. Time permitting, I will also cover Chapter 6 of the book, on robotics, and applications of algebraic geometry to computer vision, using recent papers on the subject.

1. **Elementary ring theory and Affine geometry:** Rings, ideals, zero-loci of polynomials, and a detailed study of polynomial rings in one variable.

2. **Gröbner bases:** Polynomial rings in several variables, including monomial orderings and the division algorithm. Basic properties of Gröbner bases. Hilbert Basis theorem and finite generation of ideals. Buchberger's algorithm to compute Gröbner bases; applications to ideal theory and to solving systems of polynomial equations.

3. **Elimination Theory:** Gaussian elimination beyond linear polynomials. The geometry of elimination; implicitization. Singularities and resultants.

4. **The Algebra-Geometry Dictionary:** Hilbert's Nullstellensatz. The Zariski topology of an algebraic set. Irreducible components of algebraic sets; primary decomposition of ideals.

5. **Projective Geometry:** Projective spaces and closures of affine varieties. Quadric hypersurfaces and Bezout's theorem.

**Learning Outcomes:** At the end of this course you should:

1. Be familiar with core concepts in ring theory, affine algebraic sets and the dictionary between them. These concepts include rings, ideals, ring homomorphisms, finiteness conditions on ideals, primary decompositions of ideals, and their geometric counterparts.

2. Be able to follow algorithms written in pseudo-code and to implement these algorithms using a platform such as CoCalc.
3. Understand the algorithms underpinning Gröbner bases calculations, and be able to interpret their output to deduce geometric statements about the affine algebraic sets the bases define.
4. Be capable of using symbolic calculators (e.g., CoCalc) to aide in the solution of exercises that require Gröbner bases calculations.
5. Understand the rudiments of projective geometry, and in particular the idea of adding hyperplanes at infinity to affine space in order to produce compact spaces with good intersection properties.
6. Be able to compose a concise and well-wrought abstract that serves as an informative introduction to an oral presentation.
7. Be familiar with the scientific and technical documentation system LaTeX.
8. Be able to prepare a cogent oral presentation on a mathematical theorem, with supporting slides.

**Tentative Assignment Schedule:**

August 30th: Problem Set #1 due.
September 6th: Problem Set #2 due.
September 13th: Problem Set #3 due.
September 20th: Problem Set #4 due.

**September 29th: In-class Midterm 1.**

October 4th: Problem Set #5 due.
October 11th: Problem Set #6 due.
October 18th: Problem Set #7 due.
October 25th: Problem Set #8 due.
November 1st: Problem Set #9 due.

November 5th: First draft of abstract to accompany final presentation due on Canvas (350–400 words). Slides (2-3) for short presentations due on Canvas.

Week of November 6th: Short slide presentations (2–3 slides; 5 minutes) in small groups (4–6 students), with feedback from communication coaches.

**November 15th: In-class Midterm 2.**

November 19th–21st: Individual mock presentations (10 minutes), with feedback from communication coaches. Revised, shortened abstract due (200 words).

**Final exams period: Final presentations (25 minutes) and abstracts (200 words).**