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| Instructor: | Dr. Anthony Várilly-Alvarado | Time: | MWF 9:00-9:50AM |
| Office: | 412 Herman Brown | Classroom: | Herman Brown 453 |
| Email: | varilly@rice.edu | Office Hours: | M 5:00-6:00PM, F 1:00-3:00PM |

Prerequisite knowledge: The course is aimed at advanced graduate students who are writing a thesis in algebraic geometry. Accordingly, I will assume people are comfortable with Chapters 2 and 3 of Hartshorne's text (Sections 2.1–2.8 and 3.1–3.8 should suffice; I will cover Sections 3.11 and 3.12 in the course).

Texts: *Complex Algebraic Surfaces*, Arnaud Beauville; *Algebraic Geometry*, Robin Hartshorne. I will also draw some material from *Cubic Forms*, by Yu. Manin, but I will distribute lecture notes since the book is out of print.

Homework: Due once a week, on **Fridays**. I will create a growing bank of problems of varying difficulty. You should do 2–3 problems per week and hand them in.

The homework is not pledged and you can collaborate with other students in the class. In fact, you are very much encouraged to do so. You should write up your solutions individually.

Exams: There will be no exams.

Grades: Based on homework.

Expectations: In my experience as a student, most people do not follow all the details of a Math lecture in real time. During lecture, you should expect to witness the big picture of what's going on. You should pay attention to the lecturer's advice on what is important and what isn't. A lecturer spends a long time thinking on how to deliver a presentation of an immense amount of material; they do not expect you to follow every step, but they do expect you to go home and fill in the gaps in your understanding. Not attending lecture really hurts your chances at a deep understanding of the material.

Disability Support: Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.

Topics to be covered

1. **Divisor and Linear systems:** Brief reminder of Weil and Cartier divisors, their relation to line sheaves and linear systems (Sections 2.6 and 2.7 of Hartshorne).
2. **Internal geometry of a surface:** The intersection pairing on a smooth projective surface. Riemann-Roch for surfaces; adjunction formula. Hodge Index theorem and the Nakai-Moishezon criterion for ample divisors.
3. **Rational surfaces:** Detailed study of blow-ups of the projective plane and Hirzebruch surfaces.
4. **Birational maps:** Blow-ups at a point and the structure theorem of birational maps between surfaces. The Theorem on Formal Functions and invariance of arithmetic genus under birational transformations.
5. **Castelnuovo's theorems:** Contratability of exceptional curves and numerical characterization of rational surfaces.
6. **Del Pezzo surfaces:** Their geometry and classification; lots of examples. Relation between their Picard groups and root systems.
7. **Ruled Surfaces:** Review of the global proj construction with examples. Rank 2 vector bundles over curves and their relation to ruled surfaces. The semi-continuity theorem and Grauert's theorem on higher direct images.
8. **K3 and Enriques surfaces:** Examples and basic properties. Time permitting, I will also mention examples of abelian and bi-elliptic surfaces (together with K3 and Enriques surfaces, these are the only possible kinds of algebraic surfaces with Kodaira dimension 0).
9. **Elliptic surfaces:** Examples and classification of singular fibers.
10. **Classification:** (Time permitting) Sketch of the Enriques-Kodaira birational classification via Mori theory for surfaces. A few words on surfaces of general type.