AUXILIARY FILES FOR THE PAPER "WEAK APPROXIMATION ON DEL PEZZO SURFACES OF DEGREE 1"

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Magma Scripts

The following are `Magma` scripts used for calculations in the paper. They run well in Magma v2.13 − 6. Here is a brief description of the scripts. They should be run in the order listed below.

1. `lines`: This script is designed to characterize implicitly the coefficients of the equations for the exceptional curves of the surface $w^2 = z^3 + x^6 + y^6$. Its output is used in the script `CoeffEqs`
2. `CoeffEqs`: This script computes the coefficients of the exceptional curves of the surface $w^2 = z^3 + x^6 + y^6$ using the output from the script `lines`.
3. `LinesCalculated`: In this script we put together an array called "LINES" whose entries are of the form $[z, w]$, where $z = Q(x, y)$ and $w = C(x, y)$ and $C^2 = Q^3 + x^6 + y^6$.
4. `Intersection` This script calculates the 240×240 matrix of intersections of the exceptional curves on the surface $w^2 = z^3+x^6+y^6$. It assumes the file `LinesCalculated` has been loaded. The output is stored in the matrix `IntersectionMatrix`. It takes a few seconds for this file to compile.
5. `GaloisAction` This script computes the action of the Galois Group of the splitting field of the 240 lines in the 'generic' case. The generic Galois Group has size 216 and is generated by $\langle \sigma, \tau, \phi_1, \phi_2 \rangle$. See the paper for details.
6. `WE8`: This script creates a 9-dimensional representation of $W(E_8)$ where we can embed the matrices that generated the generic Galois action in the script `GaloisAction`.
7. `Cohomology` This script computes the cohomology group $H^1(\mathrm{Gal}(K/k), \mathrm{Pic}\, X_K)$. The parameter `WhichField` near the top of the script can be adjusted to specify whether or not k contains $\sqrt[3]{2}$ and/or ζ. The script generates the output files "DataFile" and "CyclicSearchSpace" where the strategy of §3.3 has been applied and generators for $\ker \overline{N}_{L/k}$ and $\mathrm{im}\, \Delta$ are given. See the paper for more details.
8. `WarmUpExample`: These computations are useful for verifying the warm-up example of the paper. The output is a Groebner Basis for the ideal I of the example.
9. `MainTheorem`: This file contains some computations that help prove the Main Theorem. The reader is advised to understand the proof well before plunging into this file.

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9×9 MATRICES

The following 9×9 matrices correspond to σ, τ, ι_A and ι_B, respectively, as explained in §6.

$$M_\sigma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 & -1 & -1 & 2 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & 2 \\ 0 & 0 & -1 & 0 & -1 & -1 & -1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & -2 & -2 & -2 & 4 \end{pmatrix} \quad M_\tau = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & 3 \\ 0 & -1 & 0 & -1 & 0 & -1 & -1 & -1 & 2 \\ 0 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & 2 \\ 0 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & 2 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & 3 \\ 0 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & 3 \\ 0 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 3 \\ 0 & -3 & -2 & -2 & -2 & -3 & -3 & -3 & 0 \end{pmatrix}$$

$$M_{\iota_A} = \begin{pmatrix} -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & 6 \\ 0 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -2 & -2 & -2 & -1 & -2 & -2 & -2 & 5 \\ -1 & -2 & -1 & -1 & 0 & -1 & -1 & -1 & 3 \\ -1 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & 4 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & 4 \\ -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & 4 \\ -1 & -2 & -1 & -2 & -1 & -1 & -1 & -2 & 4 \\ -3 & -6 & -4 & -5 & -3 & -4 & -4 & -4 & 0 \end{pmatrix} \quad M_{\iota_B} = \begin{pmatrix} -3 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 6 \\ -2 & -3 & -2 & -2 & -2 & -2 & -2 & -2 & 6 \\ -2 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & 4 \\ -2 & -2 & -2 & -2 & -3 & -2 & -2 & -2 & 6 \\ -2 & -2 & -1 & -1 & -2 & -2 & -2 & -2 & 5 \\ -2 & -2 & -2 & -1 & -2 & -2 & -1 & -2 & 5 \\ -2 & -2 & -2 & -1 & -2 & -2 & -2 & -1 & 5 \\ -2 & -2 & -2 & -1 & -2 & -1 & -2 & -2 & 5 \\ -6 & -6 & -5 & -4 & -6 & -5 & -5 & -5 & 0 \end{pmatrix}$$

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