On this bonus assignment, credit will only be given for work which uses material covered leading up to Exam 2 – that is, up to section 4.9 of the book. You must show all your work and any necessary supporting evidence to receive full credit on a problem. An answer with no supporting work or explanation will receive little to no credit. You are allowed to use any notes, books, etcetera which you may have from class. You are not allowed to ask for help from fellow students, teachers, parents, the internet, or any other means not explicitly stated. A total of 5 points may be awarded for each problem answered completely, correctly, and with sufficient supporting work. Upon completion of this assignment, please sign the honor pledge ensuring me that these expectations have been met.

"On my honor, I have neither given nor received any unauthorized aid on this assignment."

1. (a) Show that
\[ \frac{d}{dx} \left( \frac{\sin^2(x)}{1 + \cot x} + \frac{\cos^2(x)}{1 + \tan x} \right) = -\cos 2x \]

(b) Prove that \[ \frac{d^n}{dx^n} (\sin^4 x + \cos^4 x) = 4^{n-1} \cos(4x + \frac{n\pi}{2}). \]

2. A cone of radius \( r \) centimeters and height \( h \) centimeters is lowered point first at a rate of 1 cm/s into a tall cylinder of radius \( R \) centimeters that is partially filled with water. How fast is the water level rising at the instant the cone is completely submerged?

3. (a) If \( a, b, c, \) and \( d \) are constants such that
\[ \lim_{x \to 0} \frac{ax^2 + \sin bx + \sin cx + \sin dx}{3x^2 + 5x^4 + 7x^6} = 8 \]
find the value of the sum \( a + b + c + d. \)

(b) For what value of \( a \) is the following equation true?
\[ \lim_{x \to \infty} \left( \frac{x + a}{x - a} \right)^x = e \]