Math 101 Fall 2004 Final Exam

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **three hours**. Do all 12 problems. Please do all your work on the paper provided.
Please print you name clearly here.

Print name: _____

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

_____

Grader's use only:

1. _____ /25

2. _____ /15

3. _____ /20

4. _____ /20

5. _____ /10

6. _____ /15

7. _____ /10

8. _____ /10

9. _____ /10

10. _____ /15

11. _____ /10

12. _____ /15

1. [25 points] Evaluate the derivatives of the following functions

   (a) $f(x) = \ln\left(\frac{e^x}{1+e^x}\right)$

   (b) $y = \sqrt{1 + \tan(t^2)}$

   (c) $g(x) = \arctan\left(\frac{1}{x^2}\right) + \arccos(3x^3)$

   (d) $f(t) = \int_{-\pi}^{\sqrt{t}} \cos x \, dx$

   (e) $h(z) = (4z+3)^4(z+1)^{-3}$

2. [15 points] Evaluate the following limits, if they exist.

(a) $\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

(b) $\lim_{\theta \to +\infty} \frac{\theta}{2} \sin\left(\frac{5}{\theta}\right)$

(c) $\lim_{t \to 0^+} \left(\frac{1}{t} - \frac{1}{\sqrt{t}}\right)$

3. [20 points] Evaluate the following integrals

(a) $\int_0^{\pi/2} \frac{\sin x}{(5+3\cos x)^3}\, dx$

(b) $\int \frac{x}{\sqrt{1-4x^4}}\, dx$

(c) $\int_0^1 x^2\sqrt{3-x^3}\, dx$

(d) $\int \frac{\operatorname{sech}^2(\ln x)}{x}\, dx$

4. [20 points] For the function

$$f(x) = \frac{x^3 - x^2 - 12x}{x^2 - 16},$$

the first two derivatives are

$$f'(x) = \frac{x^2 + 8x + 12}{(x+4)^2} \quad \text{and} \quad f''(x) = \frac{8}{(x+4)^3}.$$

YOU NEED NOT VERIFY THESE FORMULAS.

(a) Find all discontinuities of the function $f(x)$ and classify each as a jump, removable (point), or infinite discontinuity. For each discontinuity compute both the left and right hand limits of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing and those on which it is decreasing.

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(c) Find the critical points of $f(x)$ and classify them as local maxima, local minima or neither.

(d) Find the intervals on which $f(x)$ is concave upward and those on which it is concave downward.

5. [10 points] Compute the first three derivatives of the following function

$$g(x) = \sin(x^2)$$

6. [15 points] A billboard will cost \$1 per square foot of area plus \$10 per foot of width for the base. Given that the cost of a billboard was \$400, what is the minimum possible perimeter of the billboard? Be sure to say all the words required to justify that your answer is really the minimum.

7. [10 points] A train track runs west-east through a town and a road runs north-south through the town. A train is going east at 80 mph and a car is driving north at 50 mph. How fast is the distance between the train and the car changing when the train is 30 miles east of town and the car is 40 miles south of town?

8. [10 points] Find the area of the region in the plane bounded by

$$y = x^4 - 2x^2 \quad \text{and} \quad y = 2x^2.$$

9. [10 points] A ball is dropped from the top of a 144 ft building. Air resistance is neglected.

   (a) Solve the following initial value problem to determine the velocity function $v(t)$ and the position function $x(t)$ of the ball.

$$a(t) = -32 \text{ ft/s}^2, \quad v(0) = 0 \text{ ft/s}, \quad x(0) = 144 \text{ ft}$$

   (b) How long does it take for the ball to reach the ground? With what velocity does the ball strike the ground?

10. [15 points] Let R be the region in the plane bounded by the curve $y = x^2$ and the line $y = 3x$. Let S be the solid that results from revolving R about the y-axis. Express the volume of S as a definite integral in TWO ways, using the method of cross-sections and the method of shells. Evaluate ONE of these two integrals (your choice).

11. [10 points] Consider the curve C given by $y = x^{3/2} - \frac{1}{3}\sqrt{x}$ for $1 \leq x \leq 4$.

(a) Find the length of the curve C.

(b) Find the area of the surface that results from revolving C about the y-axis.

12. [15 points] Consider the "triangular" region R in the first quadrant between the circle $x^2 + y^2 = 9$ and the lines $x = 3$ and $y = 3$.

(a) Use geometry to compute the area of the region R.

(b) Let the centroid of the region R be denoted by the coordinates $(\bar{x}, \bar{y})$. Express $\bar{x}$ and $\bar{y}$ as definite integrals.

(c) Compute the centroid of the region R. Hint: The region R is symmetric about the line $y = x$.