

Math 101 Fall 2004 Exam 1

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 8 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.

Please print your name clearly here.

Print name: _____

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

Grader's use only:

1. _____ /10

2. _____ /15

3. _____ /10

4. _____ /20

5. _____ /10

6. _____ /10

7. _____ /15

8. _____ /10

1. [10 points] Evaluate the following limits, if they exist.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 4x + 3}$

(b) $\lim_{\theta \rightarrow 0} \frac{1}{\theta} \sin \frac{\theta}{3}$

2. [15 points] Suppose c is a constant and the function f is given by:

$$f(x) = \begin{cases} c^2x, & x < 1 \\ 3cx - 2, & x \geq 1 \end{cases}$$

- (a) Calculate $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$.
- (b) Find all values of the constant c so that the function f is continuous everywhere.

3. [10 points] (a) Give the formal, mathematical definition of the derivative of a function $f(x)$ at the point $x = a$.
- (b) Find the derivative of $f(x) = \frac{x}{1-2x}$ **using the definition of the derivative**. (No credit will be given for finding the derivative by other means.)

4. [20 points] Calculate the derivative for each of the following functions:

(a) $g(x) = (8x^2 - 5x)(13\sqrt{x} + 4)$

(b) $f(x) = \frac{\sec x}{2x^2 - 4x + 8}$

(c) $k(\theta) = \cos^2(e^{3\theta+1})$

(d) $f(x) = (7x + \ln(x^2))^6$

5. [10 points] Find the equation of the tangent line to the graph of $y = xe^{2x}$ at $x = 1$.

6. [10 points] Find the maximum and minimum value of the function $f(x) = x\sqrt{2-x^2}$ on the interval $[-\sqrt{2}, \sqrt{2}]$. Be sure to show all the steps you need to show in order to justify that your answers really are the maximum and minimum.

7. [15 points] A grain silo is to be built in the shape of a right circular cylinder with a hemispherical top. The concrete floor of the silo costs \$10 per square foot and the sides and top cost \$3 per square foot. You have 3600π dollars to spend. What is the maximum possible volume for the silo? (Some useful geometry formulas: The volume of a right circular cylinder of height h and radius r is $V = \pi r^2 h$ and the curved part of the surface of the cylinder has area $S = 2\pi r h$. The volume and surface area of a sphere of radius r are $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$, respectively.)

8. [10 points] Find dy/dx for the following by implicit differentiation.

$$(x^2 + y^2)^2 = 50xy.$$