

# Math 101 Fall 2006 Exam 1

Instructor: S. Cautis/M. Simpson/R. Stong

Thursday, October 5, 2006

*Instructions:* This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.

Please print you name clearly here.

Print name: \_\_\_\_\_

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

\_\_\_\_\_

Grader's use only:

1. \_\_\_\_\_ /15

2. \_\_\_\_\_ /10

3. \_\_\_\_\_ /25

4. \_\_\_\_\_ /10

5. \_\_\_\_\_ /15

6. \_\_\_\_\_ /15

7. \_\_\_\_\_ /10

1. [15 points] Evaluate the following limits, if they exist.

(a)  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^2 + 4x + 3}$

(b)  $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x - 4}$

(c)  $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\tan(4\theta)}$

2. [10 points] Find the derivative of  $g(t) = \frac{1}{2t+1}$  using the **definition of the derivative**. No credit will be given for derivatives obtained in any other way.

3. [25 points] Calculate the derivative for each of the following functions:

(a)  $g(x) = 1 + x^3 + e^{-x} + 5 \sin x$

(b)  $h(x) = (x + 2) \tan(6x)$

(c)  $g(\theta) = \frac{(\sec \theta)^{5/3}}{\theta}$

(d)  $f(x) = xe^{\sqrt{x}}$

(e)  $y = \ln(x^4 + x^{-2} + 3)$

4. [10 points] Find the tangent line to the curve  $y = \frac{x+1}{x-1}$  at  $(2, 3)$ .

5. [15 points] Find the maximum and minimum values of  $f(x) = \frac{e^x}{x^2+1}$  on the interval  $[0, 3]$ . Be sure to show all steps in order to justify your answers really are the minimum and maximum.

The following may be helpful  $e = 2.71828\dots$ ,  $e^2 = 7.38905\dots$ ,  $e^3 = 20.08553\dots$

6. [15 points] A cylindrical steel drum is to be made from a rectangular steel sheet and two circular sheets by bending the rectangle to form the curved portion of the cylinder and welding together the resulting seam, then welding on the two circular pieces to form the top and bottom. The total length of the welds is to be 600 in. What dimensions give the maximum possible volume?

7. [10 points] A fire has started in a dry open field and spreads in the form of a circle. The radius of the circle increases at a rate of 6 ft/min. Find the rate at which the fire area is increasing when the radius is 150 ft.