

Math 101: Midterm Exam I

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Name: _____

Problem 1(a) (5 pts)

Evaluate the following limit:

$$\lim_{\theta \rightarrow 0} \frac{\tan(\theta)}{\theta \cdot \cos^2(\theta)}$$

Carefully explain all of your work for full credit. (Hint: $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1$)

Problem 1(b) (5 pts)

Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^4 - 1}$$

Carefully explain all of your work for full credit.

Problem 2(a) (5 pts)

Demonstrate that the following limit is true via an $\epsilon - \delta$ argument:

$$\lim_{x \rightarrow 4} (5x - 15) = 5$$

Problem 2(b) (5 pts)

Apply the *Intermediate Value Theorem*, for continuous functions, in order to show that the given equation has a solution on the given interval:

$$x^3 - 2 = x \quad \text{on} \quad [0, 2]$$

Problem 3(a) (5 pts)

Use the definition of the derivative in order to compute $f'(x)$ where $f(x) = 1/(x - 2)$

Problem 3(b) (5 pts)

Compute the derivative $f'(x)$ where $f(x) = \sin(x)/\sec(x)$.

You may use any of the differentiation theorems mentioned in class.
(Hint: $\sec(x) = 1/\cos(x)$)

Problem 4 (10 pts)

Find the maximum and minimum values attained by the given function on the indicated closed interval:

$$f(x) = 3x^5 - 5x^3 \quad \text{on} \quad [-2, 2]$$

Show all of your work for full credit.

Problem 5 (10 pts)

Write an equation of the line tangent to the graph of the given function at the indicated point:

$$y = \frac{\ln x}{x^2} \quad \text{at the point} \quad (e, e^{-2})$$

Bonus Problem (10 pts)

A rocket is launched vertically upward from a point 2 miles west of an observer on the ground. What is the speed of the rocket when the angle of elevation (from the horizontal) of the observer's line of sight to the rocket is 50 degrees and is increasing at a **rate** of 5 degrees per second?