Math 101 Fall 2004 Exam 2

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.
Please print you name clearly here.

Print name: ______________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

____

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /25

4. ________ /10

5. ________ /10

6. ________ /15

7. ________ /20

1. [10 points] Compute the first three derivatives of the following function.

$$f(t) = (t^2 + 3t) \ln \left(t^2 + 3t\right)$$

2. [10 points] Evaluate the following limits, if they exist.

(a) $\lim_{t\to 0} \frac{1-\cos 3t}{t\sin t}$

(b) $\lim_{x\to 0} (1-3x)^{1/(2x)}$

3. [25 points] Evaluate the following integrals:

(a) $\int (e^t + 1)^2 \, dt$

(b) $\int x^2 \sec^2 \left(x^3 + 1\right) \, dx$

(c) $\int \sin^5 3z \cos 3z \, dz$

(d) $\int_0^1 x(2 - x^2)^3 \, dx$

(e) $\int_0^{\pi/2} e^{\sin x} \cos x \, dx$

4. [10 points] Evaluate the definite integral below directly from the definition. That is, compute $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ for a regular partition of the given interval of integration.

$$\int_0^3 (3x^2 + 1)\, dx$$

The following formulas may be helpful

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{1}{2}n^2 + \frac{1}{2}n, \quad \sum_{i=1}^{n} i^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n.$$

5. [10 points] Find the area of the region in the plane bounded by

$$y = x^3 - 3x^2 + 2x \quad \text{and} \quad y = 2x.$$

6. [15 points] We define the plane region R to be bounded by

$$y = x^2 \quad \text{and} \quad x = y^2.$$

Consider the volume V generated by rotating the region R around the x-axis.

(a) Using the method of cross-sections, compute the volume V described above.

(b) Using the method of cylindrical shells, compute the volume V described above. Note: You should get the same result as in part 6a.

7. [20 points] For the function $f(x) = \frac{\sqrt{x^2+1}}{x+5}$, the first two derivatives are $f'(x) = \frac{5x-1}{(x+5)^2\sqrt{x^2+1}}$ and $f''(x) = \frac{(3-2x)(5x^2+6x+9)}{(x+5)^3(x^2+1)^{3/2}}$. YOU NEED NOT VERIFY THESE FORMULAS.

(a) Find (and justify) all horizontal and vertical asymptotes of the graph $y = f(x)$. At any vertical asymptotes compute both the left and right hand limits of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing and those on which it is decreasing.

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(c) Find the critical points of $f(x)$ and classify them as local maxima, local minima or neither.

(d) Find the intervals on which $f(x)$ is concave upward and those on which it is concave downward. (It may be helpful to notice that $5x^2 + 6x + 9 = 4x^2 + (x+3)^2$ is positive for all x.)

(e) On the next page sketch the graph of $y = \frac{\sqrt{x^2+1}}{x+5}$ showing the results of (a)-(d). (The following values may be helpful $f(1/5) = 1/\sqrt{26} \approx 0.196$, $f(3/2) = 1/\sqrt{13} \approx 0.277$.)