Math 101, Spring 2008, Exam 2

The exam consists of 8 questions. You must show all your work to receive full credit. Please indicate your final answer clearly. Write the honor pledge on your exam when you are finished. Good luck!

In case you need them, here are some formulas:

$$\Sigma_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$
$$\Sigma_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

1) Find the limit:

a) $\lim_{t\to\infty} \frac{e^t+t^2}{e^t-t}$

b) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$

2) Compute the definite integral:

a) $\int_0^{\frac{\pi}{4}} (\sin 2x - 2\cos 5x)dx$

b) $\int_1^2 \frac{x^2}{2} - \frac{2}{x^2}dx$

3) Find the indefinite integral:

a) $\int \sec \frac{t}{3} \tan \frac{t}{3} dt$

b) $\int \frac{3dx}{x\sqrt{x}}$

4) Consider the function $f(x) = 3x + 6$ on the interval $I = [0, 4]$.

a) Using a partition of I into 4 regular subintervals, compute the right-endpoint Riemann sum for $f(x)$ associated with that partition.

b) Using the Riemann sum definition of the integral, find $\int_0^4 f(x)dx$.

5) Find the derivative of $F(x)$:

a) $F(x) = \int_0^x \sin^2 t dt$

b) $F(x) = \int_{\frac{\pi}{3}}^{e^{2x}} \sin^2 t dt$

6)

$$\begin{aligned} f(x) &= 3x^5 - 5x^3 \\ f'(x) &= 15x^2(x+1)(x-1) \\ f''(x) &= 30x(2x^2-1) \end{aligned}$$

a) Find and classify all critical points of $f(x)$ and determine the intervals on which f is increasing or decreasing.

b) Find any inflection points of $f(x)$ and determine the intervals on which f is concave up or concave down.

7)

$$f(x) = \frac{x^2+1}{x-2}$$

Find any vertical, horizontal, or slant asymptotes of $f(x)$.

8) Find $\frac{dy}{dx}$, assuming y is defined implicitly as a function of x by the equation

$$\cos x + \cos y = xy.$$

Bonus: Tell me something you have learned in another class this semester.