Rice University
Department of Mathematics
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Math 101 - Fall 2009
Midterm Exam II

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have one hour and fifteen minutes. Do all six problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. **An answer with no supporting work or explanation will receive little to no credit.**

Print Name: _____

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

_____

Question	Score
1	
2	
3	
4	
5	
6	
Total	

Problem 1(a) (5 pts)

Evaluate the following limit:

$$\lim_{x \to 0} \frac{e^x - 1}{x^3}$$

Problem 1(b) (5 pts)

Evaluate the following limit:

$$\lim_{x \to 0+} \sin x \cdot \ln x$$

Problem 1(c) (5 pts)

Evaluate the following limit:

$$\lim_{x \to \infty} (e^x + x)^{1/x}$$

Problem 2(a) (2 pts)

Consider the follwing curve:

$$y = x + \frac{4}{x^2}$$

What is the domain of this curve?

Problem 2(b) (2 pts)

What are the critical points of the curve in 2(a)?

Problem 2(c) (4 pts)

On what intervals is the curve in 2(a) increasing and decreasing? Also, identify the critical points that are either a local max or a local min.

Problem 2(d) (3 pts)

On what intervals is the curve in 2(a) concave up or concave down? Also, identify any inflection points.

Problem 2(e) (5 pts)

Identify any asymptotes for the curve in 2(a).

Hint: the curve $y = x + \frac{4}{x^2}$ can be rewritten as $y = \frac{x^3+4}{x^2}$

Problem 2(f) (4 pts)

Use all of the information that you found in the previous problems to sketch the curve in 2(a).

Problem 3 (10 pts)

Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius 3.

Hint: the image below is what it looks like to inscribe a rectangle in a quarter-circle of radius 3.

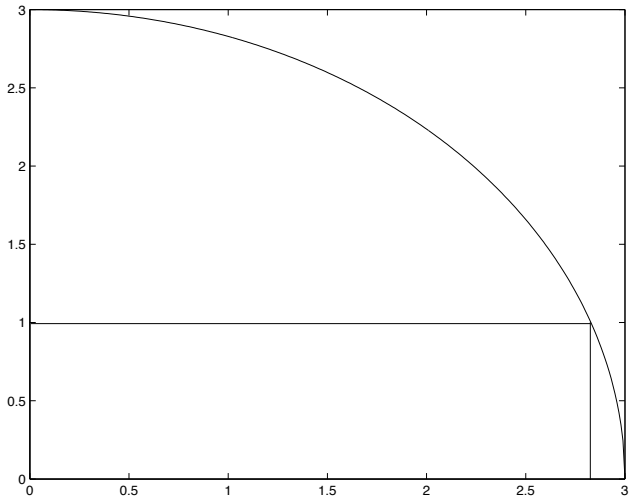

Figure 1: Rectangle Inscribed in a Quarter-Circle

Problem 4(a) (4 pts)

Show that the equation $3x + 2\cos x + 5 = 0$ has ***at least*** one real root.

Hint: use the intermediate value theorem

Problem 4(b) (6 pts)

Show that the equation $3x + 2\cos x + 5 = 0$ has ***at most*** one real root.

Problem 5 (10 pts)

Find the function $f(x)$ given the following: $f''(x) = 2 - 12x$, $f(0) = 0$, $f(1) = 1$

Problem 6 (10 pts)

The side length of a square is increasing at a rate of 8 cm/s. At what rate is the area of the square increasing when the side length of the square is 16 cm.