
Rice University
Department of Mathematics
Bridget Franklin

Math 101 - Fall 2009
Midterm Exam 1

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all **six** problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work or explanation will receive little to **no credit**.

Please print your name clearly here.

Print name: _____

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

Grader's use only:

Question	Score
1	
2	
3	
4	
5	
6	
Total	

1. Let

$$f(x) = 2 \sin 3x - 1.$$

(a) What is the domain of f ? *[2 points]*

(b) Draw the graph of f . *[6 points]*

(c) What is the range of f ? *[2 points]*

(d) State the largest interval containing 0 for which $f(x)$ is one-to-one. *[2 points]*

(e) What is $f^{-1}(0)$? *[3 points]*

2. Compute the following limits or show that they do not exist. [5 points each]

(a)

$$\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3}$$

(b)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$$

(c)

$$\lim_{\theta \rightarrow 0} \frac{\tan(6\theta)}{\sin(2\theta)}$$

3. Use the intermediate value property for continuous functions to show that the equation $e^{-x^2} = x$ has at least one solution in the interval $[0,1]$. *[8 points]*

4. (a) Write the formal, mathematical, definition of the derivative a function $f(x)$. [5 points]

- (b) Let

$$f(x) = \frac{2}{1 - 3x}.$$

Compute the derivative of $f(x)$ using the definition. No credit will be given if the derivative is found by other means. [5 points]

- (c) Find the equation of the line tangent to the graph of f at the point $(0,2)$. [5 points]

5. Compute the derivatives of the following functions. [5 points each]

(a)

$$y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$$

(b)

$$f(x) = \frac{e^x}{\sin x + x^2}$$

(c)

$$g(\theta) = (\theta^2 - 2\theta + 2) \tan \theta$$

6. Let f be the function defined by

$$f(x) = \begin{cases} x^2 \cos \frac{1}{x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

(a) Show that

$$\lim_{x \rightarrow 0} f(x) = 0.$$

(Hint: Squeeze Theorem) *[4 points]*

(b) Use the fact in (a) to argue that f is everywhere continuous. *[4 points]*

(c) Is f differentiable at $x = 0$? Justify your answer. *[4 points]*