MIDTERM SAMPLE 1

1. (a) -5 (b) 5

2. $\lim_{x\to 2^+} f(x) = 1$, $\lim_{x\to 2^-} f(x) = 1$, and $\lim_{x\to 2} f(x) = 1$, since the two side limits are equal.

f is NOT continuous at $x = 2$, because $f(2) = 1 \neq \lim_{x\to 2} f(x)$

3. $f'(x) = -\frac{1}{(x+2)^2}$.

4. $\frac{1}{12}x + \frac{4}{3}$

5. $f'(x) = \frac{3}{2\sqrt{x}} + 4x + 18x^{-4}$ $g'(x) = \frac{e^x(\sin x - \cos x + 1)}{(\sin x + 1)^2}$ $h'(t) = \frac{\sin t}{2\sqrt{t}} + \sqrt{t}\cos t$

6. Use squeeze theorem to show $\lim_{x\to 0} f(x) = 0$. As $f(0) = 0$ this proves that f is continuous at $x = 0$.
Using the definition of the derivative one gets $f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \sin\frac{1}{x}$ DNE.

7. Let $f(x) = x^3 - 6x + 3$. Then $f(-5) = -92$, $f(-2) = 7$, $f(2) = -1$, $f(5) = 98$. Apply IVP on the intervals $[-5,-2], [-2,2]$ and $[2,5]$.

MIDTERM SAMPLE 2

1. (a) 5 (b) 1/4 (c) 2

2. $\lim_{x\to 3^+} f(x) = 1$, $\lim_{x\to 3^-} f(x) = 1$, and $\lim_{x\to 3} f(x) = 1$, since the two side limits are equal.

f is NOT continuous at $x = 3$ because $f(3) = -1 \neq \lim_{x\to 3} f(x)$

3. $-\frac{3}{(3x-1)^2}$

4. $y = 4x - 2$

5. (a) $f'(x) = \frac{-3x^2+14x+31}{(x^2+4x+1)^2}$ (b) $g'(x) = \sin x(\sec^2 x + 1)$ (c) $h'(x) = \frac{1}{5}x^{-4/5}$

6. (a) Avg. velocity $= \frac{S(4)-S(0)}{4-0} = \frac{36}{4} = 9$ (b) $S'(8) = 12$ m/s

7. Let $f(x) = \frac{x}{6} + \cos x$. Then $f(0) = 1 > 0$, $f(\pi) = \frac{\pi}{6} - 1 < 0$, $f(2\pi) = \frac{\pi}{3} + 1 > 0$. Apply IVP on the intervals $[0,\pi]$ and $[\pi, 2\pi]$.

MIDTERM SAMPLE 3

1. (a) -1/2 (b) 2/5

2. (a) $\lim_{x\to 0^-} f(x) = c^2$, $\lim_{x\to 0^+} f(x) = 2c^2$. (b) The function f is continuous for $x > 0$ and $x < 0$, as it is a polynomial. For f to be everywhere continuous it needs to be continuous at 0. For this the two side limits have to be equal to $f(0)$ that is $c^2 = 2c^2 = f(0) = 2c^2$. This yields $c = 0$.

3. $\frac{1}{2\sqrt{x+3}}$

4. (a) $f'(x) = \frac{2x-1}{2x\sqrt{x}}$ (b) $g'(x) = e^x(\sin x + \cos x)$

5. $y = 2ex - e$

6. (a) Domain(f)=$\mathbb{R}$=Domain(g), Range(f)=[-1,1], Range(g)= (0,1)

(c) $(f \circ g)(x) = \sin \frac{2}{x^2+1}$, $(g \circ f)(x) = \frac{1}{\sin^2(2x)+1}$.

7. Let $f(x) = x^3 + x + 3$. Then $f(-2) = -7 < 0$ and $f(2) = 13 > 0$. Apply IVP on the interval $[-2, 2]$.