

1. Let

$$f(x) = \ln(2 + \ln x)$$

(a) find the domain of $f(x)$

(b) find the inverse function $f^{-1}(x)$ and its domain

2. Use implicit differentiation to find the equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

3. A freshly brewed cup of coffee has temperature 95°C in a 20°C room. When its temperature is 70°C , it is cooling at a rate of 1°C per minute. When does this occur?

4. Sketch the curve $y = \sqrt[3]{x^3 + 1}$. Do this by first finding the asymptotes, the critical points, the intervals of increase and decrease, the inflection points and the intervals where the graph is concave down or up.

5. A cylindrical tank with radius of 5m is being filled with water at a rate of $3\text{m}^3/\text{min}$. How fast is the height of the water increasing?

6. Evaluate the limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \sqrt{\frac{i}{n}}$$

7. Evaluate the following integrals

a. $\int \frac{x+2}{\sqrt{x^2+4x}} dx$

b. $\int_0^4 |\sqrt{x} - 1| dx$

c. $\int \frac{\sec \theta \tan \theta}{1 + \sec \theta} d\theta$

8. Consider the region \mathcal{R} bounded by the curves $y = 1/x^2$, $x = 1$, $x = 4$ and $y = 0$.

(a) Find the number a such that the line $x = a$ divides \mathcal{R} in two regions of equal area.

(a) Find the number b such that the line $y = b$ divides \mathcal{R} in two regions of equal area.

9. Use a method of your choice to compute the volume of a sphere of radius r .

10. Find the volume of the solid obtained by rotating the region bounded by $y = x^4$, $y = 0$, $x = 1$ about the line $x = 2$.
