Practice Midterm Exam #2
Math 101 – Single Variable Calculus
Summer 2007

Instructions: This is a 2 hour exam. You may not consult any notes or books during the exam. You may use a calculator on Problem 5 only. Please show your work and justify your answers.

1. **For each function, calculate the first and second derivatives.**
   (a) $f(x) = e^{2x^2+\tan x}$
   (b) $g(x) = \tan(\ln x^2)$
   (c) $h(x) = \sin^2(\sin x)$

2. **Let $y = a^x$ for a fixed positive integer a. Give a formula for $\frac{dy}{dx}$ in terms of x and a, and then prove that your formula is correct. [Hint: Use logarithmic differentiation.]**

3. **Let $y = (x^3+1)^{x^3+1}$. Use logarithmic differentiation to find $\frac{dy}{dx}$ as a function of x.**

4. **Consider the curve given by $x^2y = e^{x^3-y^2}$. Find an equation for the tangent line at $(1, 1)$. Give your equation in slope-intercept form.**

5. **Use Newton's method to approximate a solution (correct to 3 decimal places) to the equation $x^6 = x^2 + 5$ in the interval $[1, 2]$.**

   State your iterative formula and give your initial guess and the value at each step. You may use a calculator for this problem.

6. **For each of the following functions, find:**

   (i) the critical points and inflection points

   (ii) the intervals on which the function is increasing or decreasing

   (iii) the intervals on which the function is concave up or down

   (iv) the behavior of the function near the asymptotes and at infinity.

   Finally, construct a graph of each function, making sure to incorporate this information.

   (a) $f(x) = 3x^5 - 40x^3 + 105x$

   (b) $g(x) = \frac{x^2-1}{x^2+1}$

   (c) $h(x) = (x^2+1)^{3/2}$

7. **Evaluate the following limits. Identify each time you use L'Hôpital's rule and state the indeterminate form of the limit.**

   (a) $\lim_{x\to 0} \frac{\ln(x^4+1)}{\sin x^3}$

   (b) $\lim_{x\to\infty} \frac{x^3+3x^2}{\ln x^2}$

   (c) $\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{1}{\ln x^2}\right)$

   (d) $\lim_{x\to 0} \frac{e^{5x^2}-1}{x \sin x}$