

MATH 499 Exercises

January 23, 2008

1. Let Y be the cuspidal curve $y^2 = x^3$ in \mathbb{A}^2 . Blow up the point $O = (0, 0)$, and let E be the exceptional curve and \tilde{Y} be the proper transform. Show that E meets \tilde{Y} in one point, and describe an isomorphism between \tilde{Y} and \mathbb{A}^1 .
2. (i) Let Y be the cusp given by $x^3 = y^2 + x^4 + y^4$. Show that the curve \tilde{Y} obtained by blowing up Y at $O = (0, 0)$ is nonsingular. Repeat this for the node $xy = x^6 + y^6$.
(ii) A *node* is a double point (that is, a point of multiplicity 2) of a plane curve with distinct tangent directions. If P is a node on a plane curve Y and $\phi : \tilde{Y} \rightarrow Y$ is its blow-up at P , show that $\phi^{-1}(P)$ consists of two distinct nonsingular points on the blown-up curve \tilde{Y} .
(iii) Let Y be the tacnode given by $x^2 = x^4 + y^4$, and P be its singular point. If $\phi : \tilde{Y} \rightarrow Y$ is the blowing-up at P , show that $\phi^{-1}(P)$ is a node. Use (b) to show that the tacnode can be resolved by two blow-ups.
(iv) Let Y be the plane curve $y^3 = x^5$. Show that $O = (0, 0)$ is a triple point of the curve. Also, show that blowing up O produces a double point on the proper transform, and that one more blow-up resolves the singularity.