1. Let $Y$ be the cuspidal curve $y^2 = x^3$ in $\mathbb{A}^2$. Blow up the point $O = (0,0)$, and let $E$ be the exceptional curve and $\tilde{Y}$ be the proper transform. Show that $E$ meets $\tilde{Y}$ in one point, and describe an isomorphism between $\tilde{Y}$ and $\mathbb{A}^1$.

2. (i) Let $Y$ be the cusp given by $x^3 = y^2 + x^4 + y^4$. Show that the curve $\tilde{Y}$ obtained by blowing up $Y$ at $O = (0,0)$ is nonsingular. Repeat this for the node $xy = x^6 + y^6$.

(ii) A node is a double point (that is, a point of multiplicity 2) of a plane curve with distinct tangent directions. If $P$ is a node on a plane curve $Y$ and $\phi : \tilde{Y} \to Y$ is its blow-up at $P$, show that $\phi^{-1}(P)$ consists of two distinct nonsingular points on the blown-up curve $\tilde{Y}$.

(iii) Let $Y$ be the tacnode given by $x^2 = x^4 + y^4$, and $P$ be its singular point. If $\phi : \tilde{Y} \to Y$ is the blowing-up at $P$, show that $\phi^{-1}(P)$ is a node. Use (b) to show that the tacnode can be resolved by two blow-ups.

(iv) Let $Y$ be the plane curve $y^3 = x^5$. Show that $O = (0,0)$ is a triple point of the curve. Also, show that blowing up $O$ produces a double point on the proper transform, and that one more blow-up resolves the singularity.