

MATH 499 Exercises

March 19, 2008

1. For each integer $d > 0$ and each $p = 0$ or a prime number, give the equation of a nonsingular curve of degree d in \mathbb{P}^2 over a field k of characteristic p .
2. Let Y be defined by the equation $f(x, y) = 0$ in \mathbb{A}^2 , and let $P = (0, 0)$ be a point of multiplicity r on Y , so we may write $f = f_r +$ higher-order terms. Recall that a double point is a point of multiplicity two where f_2 has two distinct linear factors. More generally, we say that P is an *ordinary r -fold point* of Y if f_r is the product of r distinct linear factors.
 - (i) Show that any two ordinary double points are analytically isomorphic.
 - (ii) Show that any ordinary triple points are analytically isomorphic.
 - (iii) Show that there is a family of mutually nonisomorphic ordinary 4-fold points.
3. A homogeneous polynomial f of degree d in three variables x, y, z has $\binom{d+2}{2}$ coefficients. Let these coefficients represent a point $P \in \mathbb{P}^N$, where $N = \binom{d+2}{2} - 1 = \frac{1}{2}d(d+3)$.
 - (i) Show that there is a correspondence between points of \mathbb{P}^N and algebraic sets in \mathbb{P}^2 which can be defined by equations of degree d .
 - (ii) Show that the correspondence is one-to-one except in some cases where f has a multiple factor.