First-year grad students should do most of these problems and hand them in by next Thursday. All graduate students should do and hand in number $2, 3$ and 5, and whatever other problems would benefit them.

PROBLEM SET 2: MATH 541 FALL 2009

1. Prove that if two square integral matrices are congruent then they are cobordant.
2. Argue that if K is a knot and V is a Seifert matrix corresponding to a Seifert surface Σ for K then the reverse of K admits the Seifert matrix V^T and the mirror image admits a Seifert matrix $-V^T$. Use the same surface and the same basis. Note that you must consider how Σ gets its orientation. You must also take into account that the defintion of linking number ultimately assumes that S^3 is given the usual fixed right-handed orientation. From these facts deduce what happens to the Alexander polynomial and signatures of rK and $\overline{K}$ (compared to K itself). On a side note: if a matrix M presents a module them $-M$ presents the same module (do you see why-think gens/relations) but the transpose does not, in general, present an isomorphic module.
3. For the twist knot K_n whose bands have -1 and n twists where $n < 0$, compute the Levine-Tristram signature function. Using this computation, prove that no non-trivial integral linear combination of these knots is an algebraically slice knot. (other examples are given by torus knots but these are nastier since their genus is large).
4. Prove that the symmetric prime polynomials in $\mathbb{R}[t, t^{-1}]$ are precisely those that have all roots on the unit circle, and, except for 2 (up to units) exceptions, are quadratic.
5. Suppose K_0 and K_1 are knots whose Alexander polynomials Δ_0 and Δ_1 are relatively prime. Prove that K_0 and K_1 cannot be equal in the algebraic knot concordance group unless each knot has zero signature function and has zero value for each of the $\mathbb{Z}_2$-invariants ϕ_p as discussed in class.
6. Show that any signature $\sigma_z(K)$ (z not a root of Alexander polynomial) is an even integer. Show that
$$|\sigma_z(K)| \leq 2\text{genus}(K),$$
where the genus of K is the least genus of all Seifert surfaces.