First-year and second-year grad students should do all of these problems and hand them in by next Thursday. Other graduate students should do and hand in numbers 2, 5 and 6, and whatever other problems would benefit them.

**PROBLEM SET 3: MATH 541 FALL 2009**

1. Prove, using the geometric definition of the Blanchfield form of a knot, that the Blanchfield form is \( \Lambda \) linear in the first variable and \( \Lambda \) conjugate-linear in the second variable. Use that, since \( t \) acts by an orientation preserving homeomorphism on \( \widetilde{S^3 - K} \), \( tx \cdot ty = x \cdot y \) where \( \cdot \) is the intersection form.

2. Find knots \( K_1 \) and \( K_2 \) that have isomorphic Alexander modules, but non-isomorphic Blanchfield forms (in fact are not even equivalent in the Witt group (the knots are not equivalent in alg. concordance group). Hint: it is easy once you know the answer. Find knots \( K_3 \) and \( K_4 \) that have non-isomorphic Blanchfield forms but whose forms are equivalent in the Witt group. Finally find (seemingly) distinct knots \( K_5 \) and \( K_6 \) that have isomorphic Blanchfield forms.

3. If \((\mathcal{M}, \lambda) \in W(R, S)\) show that \((\mathcal{M}, -\lambda)\) is its inverse where \((-\lambda)(x, y) = - (\lambda(x, y))\).

4. Using the formula for the Blanchfield form in terms of the Seifert matrix, calculate \(Bl_T(1, 1)\) where the \( T \) is the right-handed trefoil knot (a twist knot) and 1 is a generator. Compare answers with friends. Is it possible that answers look different? Why?

5. Suppose \( K \) is a knot whose Alexander polynomial is not divisible by any symmetric prime. Show that \( K\) is algebraically slice.

6. Suppose that \( f : X \to Y \) is an orientation-preserving homotopy equivalence between closed, connected, oriented \( 2k \) manifolds. Show that the intersection forms on \( H_k(X; Z) \) and \( H_k(Y; Z) \) are isomorphic. Use the definition on page IV–1. You might need to look up a definition of Poincaré duality that is functorial. Deduce that \( \mathbb{CP}(2) \) is homotopy equivalent to \( -\mathbb{CP}(2) \) but not orientation-preserving homotopy equivalent.