MATH 542 FALL 2008 HOMEWORK PROBLEM SET 1

These are due in class next Thursday 9/4. You may work together. Then write them up on your own. The first two are exercises to get familiar with all of our new definitions. Try to be very rigorous with regard to the new definitions. The last problem requires more thought (I hope it's correct!).

(1) Suppose $f : M \to N$ is a homeomorphism from the topological manifold M to the smooth manifold (N, Θ) where Θ is maximal smooth atlas. Prove that there is a natural induced smooth structure on M with respect to which f is a diffeomorphism. This is called the **pulled-back smooth structure**. Start by defining an atlas on M and showing it is a smooth atlas. Say in words why the same is true if f is merely a local homeomorphism (like a covering space) (resulting only in a local diffeomorphism of course).

(2) Prove that $\mathbb{C}P(n)$ is a smooth $2n$-dimensional manifold (for those interested, almost the same proof shows it is a complex manifold of dimension n). You may assume that is is compact, Hausdorff and second countable. Recall that we can define $\mathbb{C}P(n)$ as the quotient space of $\mathbb{R}^{2n+2} - \{0\} \equiv \mathbb{C}^{n+1} - \{0\}$ using the equivalence relation $(z_0, ..., z_n) \sim (\lambda z_o, ..., \lambda z_n)$ for any $\lambda \in \mathbb{C} - \{0\}$. Do this by specifying a nice smooth atlas consisting of $n + 1$ open sets. Calculate the coordinate transformations.

(3) Prove that a surjective immersion $f : X \to Y$ between connected, compact smooth manifolds is a covering map.