MATH 542 FALL 2008 HOMEWORK PROBLEM SET 3

These are due in class next Thursday 9/18. You may work together. Then write them up on your own.

(1) You may use things proved or quoted as theorems in class: Suppose X and Y are submanifolds of a smooth manifold Z and $\dim X + \dim Y < \dim Z$. Then the map $i : X \hookrightarrow Z$ can be slightly altered so that its image is disjoint from Y. On the other hand if $\dim X + \dim Y = \dim Z$, what can you say ?

(2) The Whitney (or direct) sum of vector bundles E_0 and E_1 over B is a vector bundle over B whose fiber over x is $(E_0)_x \oplus (E_1)_x$. It is denoted $E_0 \oplus E_1$. (This is not sufficient as a definition but is adequate for this question). Recall that if M is a submanifold of N (where N has a Riemannian metric) then $N(M \hookrightarrow N)$ is defined as a subbundle of $T_M(N)$; and $T(M)$ is naturally isomorphic to a subbundle of $T_M(N)$. Show that

$$T_M(N) \cong T(M) \oplus N(M \hookrightarrow N).$$

Let ϵ^m denote any trivial bundle over B with fiber dimension m. A bundle E over B is called **stably trivial** if $E \oplus \epsilon^k \cong \epsilon^m$ for some m and k (obviously $\text{rank}(E) + k = m$). Show that the tangent bundle of S^n is stably trivial as are the tangent bundles of all the orientable surfaces. (Hint: embed). A bundle E is said to have an inverse bundle, $-E$, if $E \oplus -E \cong \epsilon^m$ for some m. Prove that the tangent bundle of any compact manifold has an inverse (same hint).