Math 542: Advanced Topics in Topology

Fall 2008 TTh 10:50-12:05 Professor Tim Cochran cochran@rice.edu

Despite the adjective "advanced", this graduate class will cover some quite basic topics in first/second year differential and algebraic topology. Knowledge of Math 444 and Math 445 will be assumed, however the requirements from homology will be slight. Grade based on attendance and some homework. There will be (almost) weekly required homework problems that will be collected and (sort of) graded. The required problems will be fairly straight-forward. More advanced students can be excused from doing problems that are, for them, redundant or too easy. Students will need to order their own books, if desired, as I have **not** ordered any through the bookstore. You don't REALLY need to buy a book for this class. That is, no book is required. I often distribute lecture notes. However, if you are going into topology or geometry then you ought to buy several!

Next semester, Professor Harvey will pick up where we have left off and possibly discuss more algebraic topics such as homology with twisted coefficients, equivariant homology, noncommutative localization of rings and modules and applications.

1. Brief Introduction to Differential Topology (approx. 2 weeks)

Smooth (differentiable) manifold, smooth structure, smooth map, tangent space, derivative of a smooth map, vector fields, Whitney Embedding theorem, Transversality.

You could just use lecture notes for this part of the class or.....

- An Introduction to Differential manifolds: Barden and Thomas (short and to the point; looks very readable; includes basic introduction to forms and de Rham cohomology but does not cover transversality. This is the book I will follow most closely for these two weeks, but only use about 50 of its 200 pages)
- Differential Topology: Morris Hirsch (the definitive text for FURTHER READING, harder to read as he does too much generality, offers little description; does contain material on vector bundles as well)
- Differential Topology: Guillemin and Pollack (popular advanced undergrad book but therefore beautifully simple and descriptive for what it covers- a little more advanced than Math 401-all manfolds are subspaces of $\mathbb{R}^n$)

2. Basic Homotopy Theory (approx. 3-4 weeks

Absolute and relative homotopy groups, role of basepoint, long exact sequences of homotopy groups fibrations, extensions of maps, Hurewicz Theorem (no proof), Whitehead Theorem, Eilenberg-Maclane spaces, basic obstruction theory.

- Lecture Notes in Algebraic Topology: James Davis and Paul Kirk, (a must-own book for a topologist)

3. Bundles and Introduction to Characteristic Classes (approx. 9 weeks

Fiber bundles and vector bundles, structure groups, tangent bundle, normal bundle, Riemannian metrics, bundle morphisms, bundle constructions, universal bundles, classification of vector bundles, introduction to characteristic classes.

- Characteristic Classes: Milnor and Stasheff (classic- must own book for a topologist- best source for vector bundles and characteristic classes)
- Lecture Notes in Algebraic Topology: James Davis and Paul Kirk, (a must-own book for a topologist-does fiber bundles)