Math 542: Advanced Topics in Topology

Fall 2013 TTh 1:00-2:15 Professor Tim Cochran cochran@rice.edu

Contrary to the title, this will class will cover some foundational topics in **Bundles and Characteristic classes**. In the smooth and other categories, much of the fine structure of manifolds and varieties is encoded in the tangent bundle or other associated bundles.

- Knowledge of Math 444 and Math 445 will be assumed.
- Grade based on attendance and several problem sets (about 6). Attendance at 67% of lectures is required to pass the course. These required homework problems that will be collected and graded. Students may consult with each other on the problems but must write up their own problems. The problems will not be difficult. Their purpose is to reinforce the material.
- required TEXTBOOK= Characteristic Classes: Milnor and Stasheff. Some copies should have been ordered by the book store. This will be supplemented by class notes.
- my office is 416 Hermann Brown Building; Tentatively I have office hours M 3-3:45, T 2:30-4pm, W 3:30-4:30 but these may change- the latest up-dates will be on my web page http://math.rice.edu/ cochran/. Normally my office hours Tuesday 3-4 pm are full of students from Linear Algebra so try to avoid this time unless you have a quick question.
- the objective of the course is for students to learn about smooth manifolds, vector bundles and characteristic classes; and to learn why they are important to the study of manifolds and other structures.
- Any student needing accommodation for a disability should see the Office of Disability Support services and then speak to me regarding their recommendation.

1. Brief Introduction to Smooth Manifolds (approx. 1-2 weeks)

This will regrettably overlap with the beginning of Professor Hassett's class last Fall, but it cannot be avoided. Smooth (differentiable) manifold, smooth structure, smooth map, tangent space, derivative of a smooth map, vector fields, Whitney Embedding theorem, transversality (?).

Other relevant texts:

- An Introduction to Differential manifolds: Barden and Thomas (short and to the point; looks very readable; includes basic introduction to forms and de Rham cohomology but does not cover transversality.
- Differential Topology: Morris Hirsch (the definitive text for FURTHER READING, harder to read as he does too much generality, offers little description; does contain material on vector bundles as well)
- Differential Topology: Guillemin and Pollack (popular advanced undergrad book but therefore beautifully simple and descriptive for what it covers- a little more advanced than Math 401-all manifolds are subspaces of $\mathbb{R}^n$)

2. Vector Bundles and Characteristic classes (approx. 10 weeks)

Vector bundles, (brief discussion of fiber bundles and principal bundles), structure groups, tangent bundle, normal bundle, tubular neighborhood theorem, Riemannian metrics, bundle morphisms, bundle constructions, orientability of bundles, universal bundles, classification of vector bundles, classification of vector bundles over spheres. Characteristic classes (Steifel-Whitney, Euler, Chern, Pontryagin).

3. Applications (approx. 1-2 weeks- as time allows)

Existence of S^7's with exotic differentiable structure. other stuff