Math 542: Advanced Topics in Topology

Spring 2012 TTh 10:50-12:05 Professor Tim Cochran cochran@rice.edu

This class will cover some basic topics in **Differential Topology**. Knowledge of Math 444 and Math 445 will be assumed, however concurrent enrollment in Math 445 will be sufficient. Grade based on attendance and some homework. There will be some required homework problems that will be collected and (sort of) graded. The required problems will be fairly straight-forward. Students will need to order their own books, if desired, as I have **not** ordered any through the bookstore. You don't REALLY need to buy a book for this class. That is, no book is required. I often distribute lecture notes. However, if you are going into topology or geometry then you ought to buy several!

1. BRIEF INTRODUCTION TO DIFFERENTIAL TOPOLOGY (APPROX. 2-3 WEEKS)

Smooth (differentiable) manifold, smooth structure, smooth map, tangent space, derivative of a smooth map, vector fields, Whitney Embedding theorem, Transversality, tubular neighborhood.

You could just use lecture notes for this part of the class or.....

- An Introduction to Differential manifolds: Barden and Thomas (short and to the point; looks very readable; includes basic introduction to forms and de Rham cohomology but does not cover transversality. This is the book I will follow most closely for these two weeks, but only use about 50 of its 200 pages)
- Differential Topology: Morris Hirsch (the definitive text for FURTHER READING, harder to read as he does too much generality, offers little description; does contain material on vector bundles as well)
- Differential Topology: Guillemin and Pollack (popular advanced undergrad book but therefore beautifully simple and descriptive for what it covers- a little more advanced than Math 401-all manifolds are subspaces of $\mathbb{R}^n$)

2. VECTOR BUNDLES (APPROX. 6-7 WEEKS

Fiber bundles and vector bundles, structure groups, tangent bundle, normal bundle, Riemannian metrics, bundle morphisms, bundle constructions, universal bundles, classification of vector bundles, classification of vector bundles over spheres, maybe brief introduction to characteristic classes.

- Characteristic Classes: Milnor and Stasheff (classic- must own book for a topologist- best source for vector bundles and characteristic classes)
- Lecture Notes in Algebraic Topology: James Davis and Paul Kirk, (a must-own book for a topologist-does fiber bundles)

3. TECHNIQUES IN DIFFERENTIAL TOPOLOGY (APPROX. 5 WEEKS

Morse theory (briefly), handlebody decompositions and theory, cobordism theory, Kirby's calculus of framed links, basic surgery theory, possibly h-cobordism theorem, Poincare Duality and intersection theory with twisted coefficients.

- Algebraic and Geometric Surgery: Andrew Ranicki
- Milnor's books on Morse theory and his Lectures on the h-cobordism theorem