examples corresponding to part 1:

2.1 problem 1:

Put $\phi(t, y, y') = 0$ into normal form, where $\phi(x, y, z) = x^2 z + (1 + x)y$. Plugging in $t = x$, $y = y$ and $z = y'$ into the expression for *phi* we get

$$\phi(t, y, y') = t^2 y' + (1 + t)y.$$

Setting this equal to zero gives the ODE

$$t^2 y' + (1 + t)y = 0.$$

subtracting $t^2 y'$ from both sides we get

$$(1 + t)y = -t^2 y'.$$

dividing by $-t^2$ this becomes

$$-\frac{(1 + t)y}{t^2} = y'.$$

Which is in normal form

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Show that the given function is a solution to the ODE

$y' = y(4 - y)$, $y(t) = \dfrac{4}{1 + Ce^{-4t}}$ for $C = 1, 2, \ldots, 5$

There is no reason to only consider these choices of C. I leave the decision to try all values of C or just the ones the problem asks for to you.

First, When is this function differentiable?

Differentiating by means of the quotient rule we get that

$$y' = \frac{-16Ce^{-4t}}{(1 + Ce^{-4t})^2}$$

whenever

$$1 + Ce^{-4t} \neq 0.$$

simplifying this we see that this choice of y is differentiable except when

$$C < 0, \text{ and } t = \ln(-C)/4$$

.

So for $C < 0$ there is no hope of this family solving the ODE on all of $(-\infty, \infty)$.

Away from these bad points, let's evaluate the right hand side (skipping the arithmetic, I don't think you need to see it)

$$y(4 - y) = \frac{-16Ce^{-4t}}{(1 + Ce^{-4t})^2}$$

.

So that, away from $C < 0$, and $t = \ln(-C)/4$, this choice of y solves the ODE.

2.2 exercise 5

solve the ODE $y' = xy + y$

This ODE is in Normal for $y'(x) = f(y(x))g(x)$ with $f(y) = y$, $g(x) = x + 1$

Solving the by substitution, when $yneq0$ we get that this ODE is equivalent to

$\frac{1}{y}y' = x + 1$

Integrating both sides with respect to x, we get

$\int \frac{1}{y}y'dx = \int x + 1dx$

Simplifying the left hand side by substitution:

$\int \frac{1}{y}dx = \int x + 1dx$

evaluating the integrals

$\ln(|y|) = \frac{1}{2}x^2 + x + cdx$ for some constant c.

exponentiating we get

$|y| = c_2e^{\frac{1}{2}x^2+x}dx$ for some positive constant $0 < c_2 = e^c$.

eliminating the absolute value bars:

$y = c_3e^{\frac{1}{2}x^2+x}dx$ for some nonzero constant $0 \neq \pm c_2 = e^c$.

Renaming constants we get the general solution

$y = ce^{\frac{1}{2}x^2+x}dx$ for any nonzero constant c.

$f(y) = y$ is zero exactly when $y = 0$ so we get the constant solution $y = 0$. This corresponds to $c = 0$ in the above formula so we get general solution

$y = ce^{\frac{1}{2}x^2+x}dx$ for any constant c.