Math 211, Differential Equations and Linear Algebra assignment 9
to be turned in by Tuesday, October 27.

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part 1:

9.4: 1, 2, 3, 20. 9.5: 10, 11, 19, 20, 53

additional problem

Those of you who performed acceptably on problem eight of the midterm can ignore this problem.

Let A and B be $n \times n$ matrices.

Part 1: Suppose that a vector v is in the nullspace of B. Show that v is in the nullspace of AB. Use this to show that if AB is nonsingular, then so is B.

Part 2: Suppose that a vector v is in the image of AB, that is $v = ABw$ for some w. Show that v is in the image of A. use this to conclude that if AB is nonsingular, then so is A.

Part 3: If both A and B are singular than there exist matrices A^{-1} and B^{-1} . Show that $(AB)(B^{-1}A^{-1}) = Id$ and $(B^{-1}A^{-1})(AB) = Id$ so that AB has an inverse matrix. Conclude that if A and B are nonsingular then AB is nonsingular.

Some presented a solution to problem 8 by arguing that nonsingularity is the same as determinant being nonzero, and then used the property that $\det(AB) = \det(A)\det(B)$. For your own benefit, you may want to understand this argument.

9.6: 10, 14, 15 19, 27

Note: We did not directly discuss at any great length how to get e^{tA} as a matrix. We instead showed how to evaluate $e^{tA}v$ for some vectors which form a spanning set. But this really is sufficient to say what the e^{At} is as a matrix. Remember that the column vectors of a matrix just encode where the standard basis vectors of $\mathbb{R}^n$ is mapped. I will write up an example.

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challenge problem:

This exercise may appeal to those of you who like to do things mechanically. This problem will outline a technique for solving high order one dimensional ODE's by means of setting up a mixing problem. If you can do this then in theory you could build an ODE solver out of some vats and pipes.

This problem has some freedom in interpretation. Describe all Linear ODE's with constant coefficients which can arise from mixing problems. The freedom is in what you interpret "mixing problem" to mean. Specify exactly what sorts of vats are allowed, and what kinds of pipes between them. The more elaborate, the better. For the sake of making things not too complicated, you may want to insist that the volume in each vat is constant. Give a description of what coefficient matrices can come from these systems.

Now that you have this information, consider what we have said about high order one dimensional linear homogeneous ODE's with constant coefficients and how they can be translated to high dimensional first order systems. Only some very specific matrices appear as coefficient matrices in this process. Which ones come from mixing problems?