example midterm problems

classify the equilibrium point at the origin and sketch a picture of solutions to the ODE $v' = Av$ for each choice of A below

1. $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$
2. $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$
3. $A = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$
4. $A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$

Most of chapter 9 all builds to solving high dimensional homogeneous linear ODEs with constant coefficients. If you can solve these you can probably do most of the contend from chapter 9.

Solve $v' = Av$ for each of the A below

1. $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

Suppose that $v_1 = \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$, and $v_1 = \begin{bmatrix} e^2 t \\ e^2 t \end{bmatrix}$

Are two linear independent solutions to $v' = Av$ with A a 2×2 matrix. Write down a funamental matrix for this ODE. Write down e^{At} as a matrix.

With the same A as immedieately above, solve $v' = Av - \begin{bmatrix} 3+t \\ t^2 \end{bmatrix}$

Solve the same inhomogeneous ODE as above using the method of undetermined coefficients.

The next topic we covered was the idea that high dimensional systems tell you something about high order ODE's. Using either this mindset directly or the techniques we got out of them, solve

$y'' + 2y' + y = 0$ $y'' + 2y' + y = e^t$ $y'' + 2y' + y = \cos(t)$ $y'' + 2y' + y = \ln(t)$

Be able to show that $e^{At}e^{Bt} = e^{(A+B)t}$ if and only if $AB = BA$

Using the series representation for e^{At} show $f(t) = e^{At}v$ solves $f' = Af$ $f(0) = v$.

More of this will likelyt be posted in the future