10.4.16: The fourth degree Taylor polynomial with remainder (also known as the fourth degree Taylor formula) for \( f(x) = \tan x \) at \( a = \frac{\pi}{4} \) is

\[
\tan x = 1 + 2 \left( x - \frac{\pi}{4} \right) + 2 \left( x - \frac{\pi}{4} \right)^2 + \frac{8}{3} \left( x - \frac{\pi}{4} \right)^3 + \frac{10}{3} \left( x - \frac{\pi}{4} \right)^4 + \int_{x}^{\pi/4} \frac{1}{24} (x-t)^4 (16 \sec^2 t \tan^4 t + 88 \sec^6 t) \, dt
\]

We can also write the formula as

\[
\tan x = 1 + 2 \left( x - \frac{\pi}{4} \right) + 2 \left( x - \frac{\pi}{4} \right)^2 + \frac{8}{3} \left( x - \frac{\pi}{4} \right)^3 + \frac{10}{3} \left( x - \frac{\pi}{4} \right)^4 + \frac{1}{120} \left( x - \frac{\pi}{4} \right)^5 (16 \sec^2 z \tan^4 z + 88 \sec^4 z \tan^2 z + 16 \sec^6 z)
\]

where \( z \) is some real number between \( \frac{\pi}{4} \) and \( x \). Either version of the remainder term is correct, and both are acceptable on your homework and on exams.

10.4.40: The Taylor series for \( f(x) = \sqrt{1+x} \) at \( a = 0 \) is

\[
1 + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-2)!}{2^{2n-1} n! (n-1)!} x^n
\]

10.8.4: The interval of convergence of this series is \((-5, 5]\).