

Even Number Solutions for Assignment 11

10.4.16: The fourth degree Taylor polynomial with remainder (also known as the fourth degree Taylor formula) for $f(x) = \tan x$ at $a = \frac{\pi}{4}$ is

$$\begin{aligned} \tan x &= 1 + 2 \left(x - \frac{\pi}{4}\right) + 2 \left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3} \left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3} \left(x - \frac{\pi}{4}\right)^4 \\ &\quad + \int_{\frac{\pi}{4}}^x \frac{1}{24} (x-t)^4 (16 \sec^2 t \tan^4 t + 88 \sec^4 t \tan^2 t + 16 \sec^6 t) dt \end{aligned}$$

We can also write the formula as

$$\begin{aligned} \tan x &= 1 + 2 \left(x - \frac{\pi}{4}\right) + 2 \left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3} \left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3} \left(x - \frac{\pi}{4}\right)^4 \\ &\quad + \frac{1}{120} \left(x - \frac{\pi}{4}\right)^5 (16 \sec^2 z \tan^4 z + 88 \sec^4 z \tan^2 z + 16 \sec^6 z) \end{aligned}$$

where z is some real number between $\frac{\pi}{4}$ and x . Either version of the remainder term is correct, and both are acceptable on your homework and on exams.

10.4.40: The Taylor series for $f(x) = \sqrt{1+x}$ at $a = 0$ is

$$1 + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-2)!}{2^{2n-1} n! (n-1)!} x^n$$

10.8.4: The interval of convergence of this series is $(-5, 5]$.