1. Neile's Parabola is not actually a parabola in the usual sense of the word, but it is one of the first transcendental curves to have its arclength computed, back in 1657 (before calculus). One parametrization for Neile's Parabola is

$$x(t) = t^2 \qquad y(t) = t^3 \qquad t \in \mathbb{R} \tag{1}$$

   (a) Sketch the graph of Neile's Parabola.

   (b) Compute its arclength from $t = 0$ to $t = 1$.

   (c) Find all non-zero values of t such that the tangent line to Neile's Parabola at $(x(t), y(t))$ passes through the point $(1, 0)$.

2. Consider the curve

$$x(t) = \cos^3 t \qquad y(t) = \sin^3 t \qquad 0 \le t \le 2\pi \tag{2}$$

   This curve is called an astroid.

   (a) Sketch the graph of the astroid.

   (b) Compute its arclength.

   (c) Compute the area enclosed by the astroid.

3. The graph of the curve

$$x(t) = t - \tanh t = t - \frac{e^t - e^{-t}}{e^t + e^{-t}} \qquad y(t) = \operatorname{sech} t = \frac{2}{e^t + e^{-t}} \qquad 0 \le t < \infty \tag{3}$$

   is called a tractrix. It is the path one gets by dragging a object originally positioned at $(0, 1)$ with a rope while walking along the positive x-axis.

   (a) Sketch the graph of the tractrix.

   (b) Compute its arclength from $t = 0$ to $t = 1$.

   (c) Find the area of the region enclosed by the positive x-axis, the graph of the tractrix, and the line segment from the origin to $(0, 1)$.

4. The Folium of Descartes (with parameter a) is a very interesting curve with parametrization

$$x(t) = \frac{3at}{1 + t^3} \qquad y(t) = \frac{3at^2}{1 + t^3} \qquad t \in \mathbb{R},\ t \neq -1 \tag{4}$$

   (a) Show that, as t approaches -1 from either side, the tangent line to the Folium of Descartes at $(x(t), y(t))$ approaches the line $y = -x - a$ (that is, the slope approaches -1 and the y-intercept approaches $-a$). Then $y = -x - a$ is an asymptote for the graph of the Folium of Descartes.

   (b) For a = 1, sketch the graph of the Folium of Descartes, along with the asymptote $y = -x - 1$.

   (c) Find the area enclosed by the curve from $t = 0$ to t approaching ∞. (Hint: you might find the substitution $u = 1 + t^3$ useful.)