

## Answers to Practice Exam 3 for the Second Midterm

1.

$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^n + \left(\frac{2}{3}\right)^n + \left(\frac{3}{4}\right)^n + \left(\frac{4}{5}\right)^n + \left(\frac{5}{6}\right)^n \right] = \boxed{20}$$

2. The third degree Taylor formula for  $f(x) = \sqrt[3]{1-x}$  at  $a = -7$  is

$$\begin{aligned} \sqrt[3]{1-x} &= 2 - \frac{1}{12}(x+7) - \frac{2}{2 \cdot 9 \cdot 32}(x+7)^2 - \frac{10}{6 \cdot 27 \cdot 256}(x+7)^3 \\ &\quad + \int_{-7}^x \frac{1}{6}(x-t)^3 \left( -\frac{80}{81}(1-t)^{-\frac{11}{3}} \right) dt \end{aligned}$$

We can also write the formula as

$$\begin{aligned} \sqrt[3]{1-x} &= 2 - \frac{1}{12}(x+7) - \frac{2}{2 \cdot 9 \cdot 32}(x+7)^2 - \frac{10}{6 \cdot 27 \cdot 256}(x+7)^3 \\ &\quad - \frac{80}{24 \cdot 81}(x+7)^4(1-z)^{-\frac{11}{3}} \end{aligned}$$

where  $z$  is some real number between  $-7$  and  $x$ . Either version of the remainder term is correct, and both are acceptable on your exam. You do not need to write both or even be familiar with both for the exam.

3. The series converges.

4. The series converges.

5. The series diverges.

6. The interval of convergence for this series is

$$\boxed{\left[ -\frac{7}{2}, -\frac{5}{2} \right]}$$