Dynamical Systems

Fall 2007 **Dr. D. Lenz**

Homework 4 **Due September 28, 2007**

(1) Consider the following differential equation in the plane

$$x' = -y + x^3 + xy^2, \quad y' = x + y^3 + x^2 y.$$

Determine the maximal solutions and find out whether they escape to infinity in finite time.

(Hint: Use polar coordinates.)

(2) Let $g : (0, \infty) \longrightarrow \mathbb{R}$ be locally Lipschitz and consider the differential equation

$$x' = -y\, g(x^2 + y^2), \;\; y' = x\, g(x^2 + y^2)$$

on $\mathbb{R}^2 \setminus \{0\}$.

(a) Determine the flow.

(b) Find a g so that all solutions are periodic with the same period.

(c) Find a g so that all orbits are periodic and different orbits have different periods.

(Hint: Use polar coordinates.)

(3) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be locally Lipschitz and assume that its support

$$K := \text{supp}(f) := \overline{\{x \in \mathbb{R}^n : f(x) \neq 0\}}$$

is a compact subset of $\mathbb{R}^n$. Show the following:

(a) The set K is invariant.

(b) f generates a global flow.

(4) Let Φ be a flow on $E \subset \mathbb{R}^n$ open. Show the following:

(a) If $M \subset E$ is invariant, so are the closure of M, the interior of M and the complement of M (in E).

(b) Every ω-limit set is closed and invariant.

(c) If $x \in E$ is attracted by $M \subset E$ (i.e. belongs to the domain of attraction of M), then $\omega(\Gamma_x) \subset \overline{M}$.