Dynamical Systems

Fall 2007 **Dr. D. Lenz**

Homework 5 **Due October 5, 2007**

(1) Let A be a real $n \times n$ matrix and ϕ the flow associated to $x' = Ax$. Show the following:

(a) A subspace U of $\mathbb{R}^n$ is invariant under A if and only if it is invariant under ϕ.

(b) A subspace U spanned by $v \neq 0$ is invariant under ϕ if and only if v is an eigenvector of A. In this case $\phi_t(u) = \exp(t\lambda)u$ for any $u \in U$, where λ satisfies $Av = \lambda v$.

(2) Let A and C be $n \times n$ matrices.

(a) Show that $t \mapsto \exp(tA)C$ solves the matrix differential equation $X' = AX$.

(b) Find neccessary and sufficient conditions (on C) such that $t \mapsto C\exp(tA)$ solves the matrix differential equation $X' = AX$.

(3) Find the stable, unstable and center subspaces for $x' = Ax$ and sketch the phase portrait for

(a) $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,

(b) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$,

(c) $A = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$,

(d) $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$.

(4) Find the stable, unstable and center subspaces for $x' = Ax$ for

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$